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The Value Distribution and Normality Criteria of a Class of Meromorphic Functions

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Abstract: In this article, we use Zalcman Lemma to investigate the normal family of meromorphic functions concerning shared values, which improves some earlier related results.

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1 Introduction and Main Results

Let D be a domain of the open complex plane $\mathbf{C}$, $f(z)$ and $g(z)$ be two nonconstant meromorphic functions defined in D, a be a finite complex value. We say that f and g share a CM (or IM) in D provided that $f-a$ and $g-a$ have the same zeros counting (or ignoring) multiplicity in D. When $a=\infty$, the zeros of $f-a$ means the poles of f (see [1]). It is assumed that the reader is familiar with the standard notations and the basic results of Nevanlinna's value-distribution theory (see [2]–[4]).

It is also interesting to find normality criteria from the point of view of shared values. In this area, Schwick[5] first proved an interesting result that a family of meromorphic functions in a domain is normal if in which every function shares three distinct finite complex numbers with its first derivative. And later, more results about shared values' normality criteria related a Hayma conjecture of higher derivative have emerged (see [6]–[13]).

Lately, Chen[14] proved the following theorems.

Theorem 1.1 *Let D be a domain in $\mathbf{C}$ and let $\mathcal{F}$ be a family of meromorphic functions in D. Let $k, n, d \in \mathbf{N}_+$, $n \geq 3$, $d \geq \dfrac{k+1}{n-2}$ and a, b be two finite complex numbers with*

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$a \neq 0$. Suppose that every $f \in \mathcal{F}$ has all its zeros of multiplicity at least k and all its poles of multiplicity at least d. If $f^{(k)} - af^n$ and $g^{(k)} - ag^n$ share the value b IM for every pair of functions $(f,\ g)$ of $\mathcal{F}$, then $\mathcal{F}$ is a normal family in D.

Theorem 1.2 *Let D be a domain in $\mathbf{C}$ and let $\mathcal{F}$ be a family of meromorphic functions in D. Let $k \in \mathbf{N}_+$ and a, b be two finite complex numbers with $a \neq 0$. Suppose that every $f \in \mathcal{F}$ has all its zeros of multiplicity at least $k+1$ and all its poles of multiplicity at least $k+2$. If $f^{(k)} - af^2$ and $g^{(k)} - ag^2$ share the value b IM for every pair of functions $(f,\ g)$ of $\mathcal{F}$, then $\mathcal{F}$ is a normal family in D.*

A natural problem arises: what can we say if $f^{(k)} - af^n$ in Theorem 1.1 is replaced by the $(f^{(k)})^m - af^n$? In this paper, we prove the following results.

Theorem 1.3 *Let D be a domain in $\mathbf{C}$ and let $\mathcal{F}$ be a family of meromorphic functions in D. Let $k, n, m, d \in \mathbf{N}_+$, $n \geq m+2$, $d \geq \dfrac{mk+1}{n-m-1}$ and a, b be two finite complex numbers with $a \neq 0$. Suppose that every $f \in \mathcal{F}$ has all its zeros of multiplicity at least $k+1$ and all its poles of multiplicity at least d. If $(f^{(k)})^m - af^n$ and $(g^{(k)})^m - ag^n$ share the value b IM for every pair of functions $(f,\ g)$ of $\mathcal{F}$, then $\mathcal{F}$ is a normal family in D.*

Theorem 1.4 *Let D be a domain in $\mathbf{C}$ and let $\mathcal{F}$ be a family of meromorphic functions in D. Let $k, m \in \mathbf{N}_+$ and a, b be two finite complex numbers with $a \neq 0$. Suppose that every $f \in \mathcal{F}$ has all its zeros of multiplicity at least $k+1$ and all its poles of multiplicity at least $mk+2$. If $(f^{(k)})^m - af^{m+1}$ and $(g^{(k)})^m - ag^{m+1}$ share the value b IM for every pair of functions $(f,\ g)$ of $\mathcal{F}$, then $\mathcal{F}$ is a normal family in D.*

2 Some Lemmas

Lemma 2.1[15] *Let $\mathcal{F}$ be a family of meromorphic functions on the unit disc satisfying all zeros of functions in $\mathcal{F}$ have multiplicity $\geq p$ and all poles of functions in $\mathcal{F}$ have multiplicity $\geq q$. Let α be a real number satisfying $-q < \alpha < p$. Then $\mathcal{F}$ is not normal at 0 if and only if there exist*

a) *a number $0 < r < 1$;*
b) *points z_n with $|z_n| < r$;*
c) *functions $f_n \in \mathcal{F}$;*
d) *positive numbers $\rho_n \to 0$*

such that $g_n(\zeta) := \rho_n^{-\alpha} f_n(z_n + \rho_n \zeta)$ converges spherically uniformly on each compact subset of $\mathbf{C}$ to a non-constant meromorphic function $g(\zeta)$, whose all zeros have multiplicity $\geq p$ and all poles have multiplicity $\geq q$ and order is at most 2.

Lemma 2.2 *Let $f(z)$ be a meromorphic function such that $f^{(k)}(z) \not\equiv 0$ and $a \in \mathbf{C}\backslash\{0\}$, $k, m, n, d \in \mathbf{N}_+$ with $n \geq m+2$, $d \geq \dfrac{km+1}{n-m-1}$. If all zeros of f are of multiplicity at least*