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Further Analysis of Global Synchronisation for Networks of Identical Cells with Delayed Coupling

Chun-Hsien Li* and Ren-Chuen Chen

Department of Mathematics, National Kaohsiung Normal University, Yanchao District, Kaohsiung City 82444, Taiwan.

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Abstract. Synchronisation is one of the most interesting collective motions observed in large-scale complex networks of interacting dynamical systems. We consider global synchronisation for networks of nonlinearly coupled identical cells with time delays, using an approach where the synchronisation problem is converted to solving an homogeneous linear system. This approach is extended to fit networks under more general coupling topologies, and we derive four delay-dependent and delay-independent criteria that ensure the coupled dynamical network is globally synchronised. Some examples show that the four criteria are not mutually inclusive, and numerical simulations also demonstrate our theoretical results.

AMS subject classifications: 05C82, 34C15, 34D06, 92B20

Key words: Synchronisation, complex networks, coupled system, time delay.

1. Introduction

Over recent years, complex coupled dynamical system networks that exhibit interesting collective motions have attracted considerable attention in various fields of science and engineering. Synchronisation is a typical collective motion in biological neural networks, where neurons couple with each other. In networks of coupled chaotic systems, synchronisation has potential application to secure communication. Refs. [5,14,24–26,30,33,39,40,42] and references therein provide more detail.

For any coupled dynamical system network, one needs to know when synchronisation is achieved, and under what conditions the synchronous state is stable. Various studies have considered complete synchronisation and criteria that guarantee its stability. For instance, conditions have been proposed using the Lyapunov function method, to assess the possibility of synchronisation via graph theory techniques [1,2], linear matrix inequality techniques [7,21,38], or an eigenvalue-based approach [15,16]. Another approach exploits the

*Corresponding author. *Email addresses:* `chli@nknucc.nknu.edu.tw` (C.-H. Li), `rcchen@nknucc.nknu.edu.tw` (R.-C. Chen)

concept of matrix measures [6, 13]. All of these articles discuss the synchronisation problem without delayed coupling, although it is well-known that information flow in complex networks is not instantaneous because the finite speed of signal transmission over distance produces a finite time delay. The influence of time delay on the dynamics should therefore be taken into account, and there are already numerous results concerning coupled dynamical networks with delayed coupling — e.g. see [4, 8, 11, 12, 18–20, 22, 23, 31, 32, 34–36, 41] and references therein. The most common approach to analysing synchronisation in delay-coupled networks invokes the Lyapunov functional method and matrix inequalities [4, 8, 9, 11, 12, 17, 20, 22, 23, 29, 34–36, 41].

In summary, many investigations of synchronisation in coupled dynamical networks with or without coupling time delays have used either a Lyapunov function or the Lyapunov functional. However, constructing a suitable Lyapunov function or Lyapunov functional for an arbitrary nonlinear system is not a trivial exercise. Moreover, even when that is possible the synchronisation criteria are often presented in the form of linear matrix inequalities that can be cumbersome to verify and restrictive. An alternative approach to the investigation of global synchronisation in coupled dynamical networks was developed by Shih & Tseng [31], where iterative arguments enable the analysis of a non-autonomous scalar equation associated with the error system such that the problem is converted into solving a corresponding homogeneous linear system. They derived both delay-independent and delay-dependent criteria for global synchronisation in the case of a ring of nonlinear delay-coupled identical cells.

In most studies of synchronisation in coupled dynamical networks, the systems are interconnected via linear coupling functions. This corresponds to neurons connected by electrical synapses in neural networks„ but neurons can also be interconnected via chemical synapses where the chemical interaction is described by a nonlinear coupling function [10]. Synchronisation in nonlinearly coupled dynamical networks is important, not only in neuroscience but also in many other fields of science and engineering due to its potential applications and theoretical interest. However, to the best of our knowledge there has so far been little progress in analysing synchronisation for the nonlinearly coupled case. Furthermore, most real-world networks display complex topologies — cf. Ref. [3, 27] and other references therein. Watts & Strogatz [37] discovered that many real-world networks display a structure that is neither random nor regular but falls somewhere in between — socalled small-world networks. The network topology can also affect the network function, but how the topological structure of a network influences its dynamical behaviour remains unclear. Consequently, global synchronisation for delay-coupled dynamical networks under nonlinear coupling function and complex network topology needs further investigation.

Employing the approach and results in Ref. [31], we improve the synchronisation criteria to fit networks under more general coupling topologies, and find both delay-dependent and delay-independent criteria ensuring global synchronisation. These criteria involve both the intrinsic parameters of the isolated cells and coupling time delays, and we find that the network topology can specifically affect the synchronisation behaviour. In Section 2, we introduce the coupled dynamical network and briefly summarise results deduced from Ref. [31]. In Section 3, we derive our four criteria to ensure the occurrence of global syn-