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A Blow-up Result with Arbitrary Positive Initial Energy for Nonlinear Wave Equations with Degenerate Damping Terms

SU Xiao[1,*] and WANG Shubin[2]

[1] *College of Science, Henan University of Technology, Zhengzhou 450001, China.*
[2] *School of Mathematics and Statistics, Zhengzhou University, Zhengzhou 450001, China.*

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Abstract. This article is concerned with the finite time blow-up of weak solutions to the wave equations with nonlinear damping and source terms. We provide the sufficient conditions of finite time blow-up of weak solutions with arbitrary positive initial energy by constructing a energy perturbation function.

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1 Introduction

This paper is devoted to the finite time blow-up of weak solutions to the following problem

$$u_{tt}-\Delta u+|u|^k\partial j(u_t)=|u|^{p-1}u \quad \text{in } \Omega\times(0,T) \tag{1.1}$$

$$u=0 \quad \text{on } \partial\Omega\times(0,T) \tag{1.2}$$

$$u(x,0)=u_0(x)\in H_0^1(\Omega), \quad u_t(x,0)=u_1(x)\in L^2(\Omega) \tag{1.3}$$

where $1<p<\infty$, $k\geq 0$, Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, $j(s)$ is a continuous, convex real-valued function defined on $\mathbb{R}$ and $\partial j(s)$ is its sub-differential [1].

The solvability of the problem (1.1)-(1.3) has been studied by Barbu et al. [2], in which the local generalized, weak and strong solutions were obtained, respectively under some

*Corresponding author. *Email addresses:* `suxiao@haut.edu.cn` (X. Su), `wangshubin@zzu.edu.cn` (S. Wang)

assumptions on $j(s)$ and the parameters in the equations. In addition, with further restriction on the parameters, the authors proved that the local generalized and weak solutions are global whenever $p \le k+m$ and the weak solutions blow up in a appropriate time provided that $p > k+m$, and the initial energy is negative. Later, Barbu et al. [3] proved that the generalized solutions enjoying an additional regularity blow up in a finite time provided $p > k+m$, and the initial energy is negative. Hu [4] addressed the issues of finite time blow-up of solutions with the small positive initial energy. Rammaha [5] proved that weak solutions blow up in finite time whenever the initial energy is negative and the exponent of the source term is more dominant than the exponents of both damping terms for the systems of (1.1).

Taking $j(s) = \frac{1}{m+1}|s|^{m+1}$, the equation (1.1) becomes

$$u_{tt} - \Delta u + |u|^k |u_t|^{m-1} u_t = |u|^{p-1} u, \tag{1.4}$$

which is the well-known polynomially-damped wave equation. Pitts and Rammaha [6] studied the existence and nonexistence of global weak solutions of the initial-boundary value problem of (1.4) with $0 < m < 1$ and $p,k > 1$. The results on global weak solutions and finite time blow-up of weak solutions of the initial-boundary value problem of (1.4) with $m = 1$ in one space dimension are also established by Rammaha [7]. Benaissa [8] considered the blow up results of solutions when the initial energy has an positive upper bound for the systems of (1.4).

If $k = 0$, the equation (1.4) then reads

$$u_{tt} - \Delta u + |u_t|^{m-1} u_t = |u|^{p-1} u, \tag{1.5}$$

which can be treated via perturbation theory of monotone operators [1]. Georgiev and Todorova [9] considered the equation (1.5) with the conditions (1.2) and (1.3), and it was shown that if $1 < p < \infty$ for $n = 1,2$ and $1 < p \le \frac{n}{n-2}$ for $n \ge 3$, then all finite energy solutions are global when $p \le m$, while if $m < p$, then solutions with sufficiently large negative initial energies blow up in finite time, whose method is later extended to accommodate other damped hyperbolic like dynamics [2,6]. Todorova [10] studied the global well-posedness for the $H^1(\mathbb{R}^3)$-critical cases when $p=5$ and $m=\frac{2}{3}$. For the systems of (1.5), Rammaha [11] considered the blowup of solutions with negative initial energy. Houari [12] studied the global nonexistence of solutions with positive initial energy and certain class of initial data. Peravi [13] obtained the lower bounds for the blow up time. As far as I know, there is no results about the finite time blow-up of weak solutions to (1.1)-(1.3) with arbitrary positive initial energy.

The purpose of this paper is to provide sufficient conditions of blow-up of weak solutions to (1.1)-(1.3) with arbitrary positive initial energy. It means that for any constant $K > 0$, we can construct infinitely many initial data from the energy space with the initial energy $E(0) = K$ such that the weak solutions blow up in finite time.