On the Uniqueness of Traveling Forced Curvature Fronts in a Fibered Medium

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Abstract. We investigate traveling fronts, including pulsating ones, of a forced curvature flow in a plane fibered medium. The main topic of this note is an uniqueness issue of such traveling fronts. In addition to line-shaped profiles, we also consider traveling fronts in the form of V-shaped parabolas.

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1 Introduction

In this note, we will be interested in traveling fronts of a forced curvature flow equation

\[ V_n = R + K \]  \hspace{1cm} (1.1)

in the plane containing periodic striations. \( V_n \) is the normal velocity of a propagating interface \( \Gamma(t) \), \( K \) is its mean curvature and \( R \) is the driving force. For example if \( \Gamma \) is a flame front, then \( R \) corresponds to the combustion rate of the burning material. In all cases, we will suppose that the function \( R \) is smooth and verifies

\[ 0 < R_m \leq R \leq R_M. \]  \hspace{1cm} (1.2)

Before going further, let us give a definition of a traveling front of Eq. (1.1).

Definition 1.1. \( \Gamma(t) \), solution of (1.1) will be called a traveling front if there exists a constant vector \( v \in \mathbb{R}^2 \) such that

\[ \Gamma(t) = \Gamma_0 + v \cdot t \]

for all \( t \in \mathbb{R} \). Then \( \Gamma_0 \) is the (constant) profile of the traveling front and \( |v| \), its speed, see Figure 1.

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Note that if $\Gamma(t)$ can be represented by the graph of a function $u$ in the $x$-$y$ plane, for example
\[ \Gamma(t) = \{(x,y)/y = u(x,t)\}, \]
then $V_n$ is given by
\[ V_n = \frac{u_t}{\sqrt{1+u_x^2}}, \]
so that Equation (1.1) becomes
\[ u_t - R \sqrt{1+u_x^2} = \frac{u_{xx}}{1+u_x^2}, \quad t \in \mathbb{R}, x \in \mathbb{R}. \tag{1.3} \]

Now if $\Gamma(t)$ is a traveling front in the plane, we can suppose without loss of generality that $v$ is parallel to the $y$-axis i.e. $v = \ell(0,t)$. Then $u(x,t)$ will be given by
\[ u(x,t) = c + \varphi(x), \]
so that Equation (1.3) becomes
\[ c - R \sqrt{1+\varphi_x^2} = \frac{\varphi_{xx}}{1+\varphi_x^2}, \quad x \in \mathbb{R}. \tag{1.4} \]

In the above, $c$ is the speed and $\varphi$ the constant profile of the wave. The pair $(c,\varphi)$ will be called a traveling wave solution (TWS) of Eq. (1.3). Note that every solution $\varphi$ of (1.4) is defined up to an additive constant.

Figure 1: A TWS: a constant profile moving with a constant speed in some given direction.