Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2016.m1428

Vol. **9**, No. 2, pp. 169-184
May 2016

One-Bit Compressed Sensing by Greedy Algorithms

Wenhui Liu, Da Gong and Zhiqiang Xu*

LSEC, Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.

Received 9 November 2014; Accepted 7 April 2015

Abstract. Sign truncated matching pursuit (STrMP) algorithm is presented in this paper. STrMP is a new greedy algorithm for the recovery of sparse signals from the sign measurement, which combines the principle of consistent reconstruction with orthogonal matching pursuit (OMP). The main part of STrMP is as concise as OMP and hence STrMP is simple to implement. In contrast to previous greedy algorithms for one-bit compressed sensing, STrMP only need to solve a convex and unconstrained subproblem at each iteration. Numerical experiments show that STrMP is fast and accurate for one-bit compressed sensing compared with other algorithms.

AMS subject classifications: 94A12; 65H99; 65D15.

Key words: Compressed sensing, sparse signals, greedy algorithm.

1. Introduction

Compressed sensing, or compressive sensing provides a new method of data sampling and reconstruction, which allows to recover sparse signals from much fewer measurements [5,7]. Suppose that we have an unknown sparse signal $\hat{x} \in \mathbb{R}^n$ with $\|\hat{x}\|_0 \le s$ and $s \ll n$, where $\|\cdot\|_0$ denotes the number of nonzero components. We observe the signal as

$$b = A\hat{x},$$

where $A \in \mathbb{R}^{m\times n}$ is called measurement matrix, $b \in \mathbb{R}^m$ is the vector of measurements. Compressed sensing shows that only $m = O(s\log(n/s))$ measurements are sufficient for exact reconstruction of $\hat{x}$ under many settings for the measurement matrix A [6,7].

1.1. One-Bit compressed sensing

In compressed sensing, it is supposed that the measurements have infinite bit precision. However, in practice what we get is quantized measurements. In other words, the entries in the measurement vector b must be mapped to a discrete set of values

*Corresponding author. *Email addresses:* liuwenhui11@mails.ucas.ac.cn (W. -H. Liu), gongda@lsec.cc.ac.cn (D. Gong), xuzq@lsec.cc.ac.cn (Z. -Q. Xu)

$\mathcal{A}$. There are much work about the recovery of the general signal from the quantized measurements [18]. In this paper, we focus on the case where $\mathcal{A} = \{-1, 1\}$ with the mapping being done by the sign function. So we need to recover an s-sparse signal from $y := \mathrm{sign}(b) \in \{-1, 1\}^m$. This problem is called one-bit compressed sensing, which was first introduced by Boufounos-Baraniuk [4]. In one-bit compressed sensing, we observe original signal as:

$$y = \mathrm{sign}(A\hat{x}),$$

where $y \in \mathbb{R}^m$ with each element is sign of the corresponding element of $A\hat{x}$. That means we lost all magnitude information of $A\hat{x}$. Following [8], $x^\sharp \in \mathbb{R}^n$ is called *a solution for one-bit compressed sensing corresponding to A and $\hat{x}$* if it satisfies

(i) consistence, i.e. $\mathrm{sign}(Ax^\sharp) = \mathrm{sign}(A\hat{x})$,

(ii) sparsity, i.e. $\|x^\sharp\|_0 \le \|\hat{x}\|_0$.

A simple observation is that $\mathrm{sign}(Ac\hat{x}) = \mathrm{sign}(A\hat{x})$ and $\|c\hat{x}\|_0 \le \|\hat{x}\|_0$ where $c > 0$ is a scale. Thus the best one-bit compressed sensing can do is to recover $\hat{x}$ up to a positive scale. Therefore, we usually expect to recover original signal on the unit Euclidean sphere.

1.2. Previous work

A straightforward way to obtain a solution for one-bit compressed sensing is to solve the following program:

$$\begin{aligned} &\min\ \|x\|_0 \\ &\text{s. t. } y = \mathrm{sign}(Ax) \ \text{ and } \ \|x\|_2 = 1. \end{aligned} \tag{1.1}$$

Since (1.1) is computational intractable, similar with compressed sensing, one can replace the ℓ_0 norm by the more tractable ℓ_1 norm and obtain that (see [4, 9, 10])

$$\begin{aligned} &\min\ \|x\|_1 \\ &\text{s. t. } y = \mathrm{sign}(Ax) \ \text{ and } \ \|x\|_2 = 1. \end{aligned} \tag{1.2}$$

Many algorithms have been proposed to solve (1.2). Particularly, in [9], Laska et. al. use the augmented Lagrangian optimization framework to design RSS algorithm by employing a restricted-step subroutine to solve a non-convex subproblem. Binary iterative hard thresholding (BIHT) and adaptive outlier pursuit (AOP) are introduced in [8] and [12], respectively. BIHT is a modification of iterative hard thresholding which is to solve compressed sensing problem (see [2]). AOP is a robust algorithm built on BIHT, and it is exactly BIHT when measurements are noise free. The numerical experiments in [12] show that AOP performs better than the previous existing algorithms in terms of the recovery performance. In [10], Plan and Vershynin replace the normalization constraint $\|x\|_2 = 1$ by $\|Ax\|_1 = c_0$ and give an analysis of the following convex