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A Globally and Superlinearly Convergent Primal-dual Interior Point Method for General Constrained Optimization

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Abstract. In this paper, a primal-dual interior point method is proposed for general constrained optimization, which incorporated a penalty function and a kind of new identification technique of the active set. At each iteration, the proposed algorithm only needs to solve two or three reduced systems of linear equations with the same coefficient matrix. The size of systems of linear equations can be decreased due to the introduction of the working set, which is an estimate of the active set. The penalty parameter is automatically updated and the uniformly positive definiteness condition on the Hessian approximation of the Lagrangian is relaxed. The proposed algorithm possesses global and superlinear convergence under some mild conditions. Finally, some preliminary numerical results are reported.

AMS subject classifications: 65K05, 90C30, 90C31

Key words: general constrained optimization, primal-dual, active set, global convergence, superlinear convergence.

1. Introduction

Consider the following general constrained optimization problem

$$
\text{(P)}\quad \begin{array}{ll} \min & f_0(x) \\ \text{s.t.} & f_i(x)\le 0,\quad i\in I=\{1,2,\dots,m\}, \\ & f_i(x)=0,\quad i\in L=\{m+1,m+2,\dots,m+l\}, \end{array}
$$

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where functions $f_0, f_i\ (i \in I \cup L) : R^n \to R$ are all continuously differentiable.

It is well known that sequential quadratic programming (SQP) methods is one of efficient methods for constrained optimization. The interested readers are referred to Boggs et al. [1] and Gill et al. [5] for more information on SQP methods. However, SQP methods have to solve QP subproblems at each iteration, which is computationally expensive. Panier et al. in [10] presented a feasible QP-free algorithm (Algorithm PTH for short) for (P) only with inequality constraints. At each iteration, only three systems of linear equations (SLEs) need to be solved. Specifically, for the current iterative point x^k, the master search direction d^k is generated by solving two SLEs in (d, λ) with the following form

$$\begin{pmatrix} H_k & A_k \\ Z_k A_k^{\mathrm{T}} & G_k \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -\nabla f_0(x^k) \\ \eta \end{pmatrix}, \tag{1.1}$$

where H_k is an approximation of the Hessian of the Lagrangian associated with (P), the vector η is chosen in different ways, $A_k = (\nabla f_i(x^k), i \in I)$, $G_k = \mathrm{diag}(f_i(x^k), i \in I)$, $Z_k = \mathrm{diag}(z_i^k, i \in I)$ and $z_i^k\ (i \in I)$ the current estimate of the KKT multipliers. In order to avoid the Maratos effect, the search direction is modified by solving a least squares subproblem, which is equivalent to an SLE. The global convergence of Algorithm PTH requires a strong assumption, i.e., the number of stationary points is finite.

Algorithm PTH was later improved by Gao et al. in [6]. An extra SLE is solved and the algorithm globally converges to a KKT point under some conditions including a strong assumption that the multiplier sequence is bounded, which is impossible if the iteration matrix is ill-conditioned. Afterwards, based on the Fischer-Burmeister function and the KKT nonsmooth equation system, Qi and Qi in [12] also improved Algorithm PTH and proposed a new feasible QP-free algorithm. At each iteration, three SLEs, which differ from those in [10], are required to solve. It is shown that the coefficient matrix of the SLE is uniformly nonsingular and the multiplier approximation sequence is bounded, even if the strict complementarity is not satisfied. Without assuming the isolatedness of the stationary points, the global convergence is proved under some suitable conditions.

As we know, primal-dual interior point methods are also one of efficient solution methods for problem (P) (see, e.g., [2,11,13,15]). Especially, Bakhtiari and Tits in [2] proceeded along the lines of Algorithm PTH and proposed a simply feasible primal-dual interior-point method (Algorithm BT for short). For completeness, we first give a short introduction about feasible primal-dual interior point (PDIP) methods.

The KKT system of problem (P) only with inequality constraints is given as follows

$$\begin{cases} \nabla_x L(x, \lambda) = 0, \\ \lambda_i f_i(x) = 0,\ \lambda_i \geq 0,\ f_i(x) \leq 0,\ i \in I, \end{cases} \tag{1.2}$$

where the Lagrangian function is given by $L(x, \lambda) = f_0(x) + \sum_{i \in I} \lambda_i f_i(x)$. The key point of PDIP iteration is a perturbed variant of (1.2) in (x, λ) as follows

$$\begin{cases} \nabla_x L(x, \lambda) = 0, \\ \lambda_i f_i(x) = \mu_i,\ i \in I. \end{cases} \tag{1.3}$$