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Numerical Schemes for Linear and Non-Linear Enhancement of DW-MRI

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Abstract. We consider the linear and non-linear enhancement of diffusion weighted magnetic resonance images (DW-MRI) to use contextual information in denoising and inferring fiber crossings. We describe the space of DW-MRI images in a moving frame of reference, attached to fiber fragments which allows for convection-diffusion along the fibers. Because of this approach, our method is naturally able to handle crossings in data. We will perform experiments showing the ability of the enhancement to infer information about crossing structures, even in diffusion tensor images (DTI) which are incapable of representing crossings themselves. We will present a novel non-linear enhancement technique which performs better than linear methods in areas around ventricles, thereby eliminating the need for additional preprocessing steps to segment out the ventricles. We pay special attention to the details of implementation of the various numeric schemes.

AMS subject classifications: 65M10, 78A48

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1. Introduction

Diffusion-Weighted Magnetic Resonance Imaging (DW-MRI) is an MRI techniques for non-invasively measuring local water diffusion inside tissue. It has been stipulated that the water diffusion profiles of the imaged area allow inference of the underlying tissue structure. For instance in brain white matter, diffusion is less constrained parallel to nerve fibers

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than perpendicular to them and so the water diffusion gives information about the fiber structures present. This allows for the extraction of anatomical information concerning biological fiber structures from DW-MRI scans.

The diffusion of water molecules in tissue over some time interval t can be described by a diffusion propagator which is the probability density function $\mathbf{x} \mapsto p_t(X_t = \mathbf{x}+\mathbf{r} \mid X_0 = \mathbf{x})$ of finding a particular water molecule at time $t \geq 0$ displaced by $\mathbf{r} \in \mathbb{R}^3$ from its initial position $\mathbf{x} \in \mathbb{R}^3$ at $t = 0$. Here the family of random variables $(X_t)_{t \geq 0}$ describes the distribution of water molecules over time. The function p_t can be related to MRI signal attenuation of diffusion weighted image sequences through the Fourier transform and so can be estimated given enough measurements [28]. The exact methods to do this are described by e.g., Alexander [2].

The most common DW-MRI scans is Diffusion Tensor Imaging (DTI), which measures for each position a 3×3, symmetric positive definite tensor called the diffusion tensor [3]. DTI makes the assumption that locally the diffusion propagator is given by an anisotropic Gaussian function, characterized by this diffusion tensor. Other, more recent techniques collectively called High Angular Resolution Diffusion Imaging (HARDI), allows more general shapes for the diffusion propagator [8, 32].

Often, p_t is not reconstructed completely from measurements, since this would involve long scanning times. Instead, the Orientation Distribution Function(ODF) can be obtained using less measurements [1]. The ODF gives the probability density that a water particle diffuses in a certain direction $\mathbf{n}$, regardless of distance traveled. It is defined by:

$$\mathrm{ODF}(\mathbf{x}, \mathbf{n}) = \int_0^\infty p_t(X_t = \mathbf{x} + \alpha \mathbf{n} \mid X_0 = \mathbf{x}) \alpha^2 d\alpha. \tag{1.1}$$

This ODF is an example of a function of position and orientation. A general function $U : \mathbb{R}^3 \times S^2 \mapsto \mathbb{R}^+$ of positions and orientations can be visualized by a field of surfaces

$$S_\mu(U)(\mathbf{x}) = \{\mathbf{x} + \mu U(\mathbf{x}, \mathbf{n})\mathbf{n} \mid \mathbf{n} \in S^2\} \subset \mathbb{R}^3, \tag{1.2}$$

which are called glyphs. A figure is generated by visualizing all these surfaces for different, sampled $\mathbf{x}$ and with a suitable value for $\mu > 0$ that determines the size of the glyphs. An example of such a visualization can be found in the top left of Fig. 1. Note that for DTI data a different visualization based on ellipsoids is commonly used. Such an ellipsoid visualization can not handle our enhanced DW-MRI images, which can represent crossings, which explains our choice for visualization. To reduce noise and to infer information about fiber crossings, contextual information can be used [25, 26]. This enhancement is useful both for visualization purposes and as a preprocessing step for other algorithms, such as fiber tracking algorithms, which may have difficulty in noisy or incoherent regions. Recent studies indicate the increasing relevance for enhancement techniques in clinical applications [5, 19, 21, 30, 31].

We perform the enhancements on modeled DWI images. We use two models: diffusion tensors and orientation distribution functions. We want to stress that the algorithms can be applied to any other model, as long as it can be converted to a function of positions and orientations. We will use the term DWI data in this paper to refer to both these models.