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Wavelets and Optical Flow Motion Estimation

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Abstract. This article describes the implementation of a simple wavelet-based optical-flow motion estimator dedicated to continuous motions such as fluid flows. The wavelet representation of the unknown velocity field is considered. This scale-space representation, associated to a simple gradient-based optimization algorithm, sets up a well-defined multiresolution framework for the optical flow estimation. Moreover, a very simple closure mechanism, approaching locally the solution by high-order polynomials is provided by truncating the wavelet basis at fine scales. Accuracy and efficiency of the proposed method is evaluated on image sequences of turbulent fluid flows.

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1. Introduction

Recent years have seen significant progress in signal processing techniques for fluid motion estimation. The wider availability of image-like data, whether coming from experimental facilities (e.g., particle image velocimetry) or from larger-scale geophysical study systems such as lidars or meteorological and oceanographical satellites, strongly motivates the development of computer-vision methods dedicated to their analysis. Correlation-based and variational methods have proven to be efficient in this context. However, the specific nature of fluid motion highly complicates the process. Indeed, one has to deal with continuous fields showing complex structures evolving at high velocities. This is particularly problematic with optical flow methods, where the problem non-linearity requires to resort to an ad-hoc multiresolution strategy. Although leading to good empirical results, this technique is known to have a number of drawbacks. Moreover, the underdetermined nature of the optical flow estimation problem imposes to add some prior information about

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the sought motion field. In many contributions dealing with rigid-motion estimation, first-order regularization is considered with success. However, when tackling more challenging problems such as motion estimation of turbulent fluids, this simple prior turns out to be inadequate. Second-order regularizers [5,16,17], or a first order regularizer [7] allowing to enforce physically-sound properties of the flow, are considered; but their implementation raises up several issues.

In this paper, we propose an optical flow estimation procedure based on a wavelet expansion of the velocity field. This approach turns out to offer a nice mathematical framework for multiresolution estimation algorithms, which avoids some of the drawbacks of the usual approach. Note that algorithms based on wavelet expansion of the data [1], of the velocity field [15] or even both [4] have been previously proposed. However, unlike the algorithm presented hereafter, their computational complexity and/or lack of multiscale mechanism significantly limits their application to small images and/or the estimation of the coarsest motion scales in [15], and might raise up issues when dealing with large displacements in [4]. Finally, the multiscale wavelet framework also suggests a very simple regularization by neglecting smallest scales coefficients; it turns out to be particularly adapted to "smooth enough" motions.

This article processes as follows: Section 2 recalls concepts behind optical flow estimation. Sections 3 and 4 introduce the wavelet framework, then describe its integration into the optical flow problem and the implementation of the resulting algorithm. Behavior and efficiency of the proposed estimator are finally assessed in Sections 5 and 6, using both synthetic and real flow visualization images.

2. Optic flow problem

The optical flow problem consists in estimating the apparent 2D displacement within a 3D scene depicted by a sequence of images, e.g., obtained from a camera. The time- and space-variations of an observable image quantity, e.g., its brightness, are used to infer the underlying motion. In the following, we denote by $I(\boldsymbol{x},t)$ the brightness of the image at pixel $\boldsymbol{x} \in \Omega$, with $\Omega \subset \mathbb{R}^2$ the image domain, and at a discrete time $t \in \mathbb{N}$. The optical flow, as a 2D vector field $\boldsymbol{v}(\boldsymbol{x},t) : \Omega \times \mathbb{N} \mapsto \mathbb{R}^2$, is the projection on the image plane of the actual 3D motion. It is a *dense* field, since it provides one velocity vector per pixel of the input images. The optical flow estimation involves two main aspects: the *data term*, which links the motion $\boldsymbol{v}$ to be estimated to the input data — here, image brightness I —, and a *regularization mechanism* to overcome the ill-posedness of the problem.

2.1. Data term

Data terms are built upon assumptions on the behavior of the observed image quantity. The most simple and most widely used is a conservation assumption of the image brightness:

$$\frac{dI}{dt}(\boldsymbol{x},t) = \frac{\partial I}{\partial t}(\boldsymbol{x},t) + \boldsymbol{v}(\boldsymbol{x},t) \cdot \nabla I(\boldsymbol{x},t) = 0. \tag{2.1}$$