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A Primal-Dual Hybrid Gradient Algorithm to Solve the LLT Model for Image Denoising

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Abstract. We propose an efficient gradient-type algorithm to solve the fourth-order LLT denoising model for both gray-scale and vector-valued images. Based on the primal-dual formulation of the original nondifferentiable model, the new algorithm updates the primal and dual variables alternately using the gradient descent/ascent flows. Numerical examples are provided to demonstrate the superiority of our algorithm.

AMS subject classifications: 68U10, 65K10

Key words: LLT model, image denoising, primal-dual.

1. Introduction

Image denoising is one of the most fundamental tasks in image processing. For generality, let us consider a vectorial function $\mathbf{u}(x)$ defined on a bounded domain $\Omega \subset \mathbb{R}^2$:

$$\begin{aligned} &\mathbf{u}: \ \Omega \to \mathbb{R}^d \\ &x = (x_1, x_2) \mapsto \mathbf{u}(x) := (u_1(x), u_2(x), \cdots, u_d(x)). \end{aligned} \tag{1.1}$$

The observed noisy image $\mathbf{u}_0(x)$ is modeled as

$$\mathbf{u}_0 = \mathbf{u} + \mathbf{n}, \tag{1.2}$$

where $\mathbf{n}$ is a d-dimensional additive Gaussian noise with zero-mean. When $d = 1$ and 3, we get the model for gray-scale and color images, respectively. The task of image denoising is to recover the true image $\mathbf{u}$ from the given noisy data $\mathbf{u}_0$, which belongs to the inverse problems. The well-known ill-posedness of inverse problems is often handled by some regularization methods. Tikhonov type regularization is very popular and useful for many

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types of inverse problems. In the field of image denoising, one of the most famous and influential approaches is the ROF model, which is proposed by Rudin, Osher, and Fatemi in [15] originally for gray-scale image denoising. The ROF model aims to minimize the following energy functional:

$$\int_\Omega |\nabla u(x)|dx + \frac{\beta}{2}\int_\Omega |u-u_0|^2 dx, \tag{1.3}$$

where the first term is the total variation (TV) term and $\beta > 0$ is a parameter that controls the contribution of the fidelity term [5]. The novelty of the ROF model is that it allows discontinuous solutions and hence can preserve sharp edges while removing noise. The vectorial TV can be defined similarly and has been applied to some color image processing problems [1,2].

However, an undesired disadvantage of the TV based model is the "staircase" effects [12,13], as can be seen from images in [15,18], which is the result of piecewise constant solutions. A variety of higher order variational models (especially fourth-order) are introduced to alleviate the staircase effects [6,8,11,13,16,21]. In [13], a new fourth-order filter based variational model, which is referred as the LLT model, has been proposed. To better preserve edges and avoid the oversmoothing of the fourth order models simultaneously, some mixed models have been proposed [12,14]. Lysaker and Tai [14] devise an iterative scheme by a convex combination of the results of the second and the fourth order models. A variational approach combining a total variational filter and a fourth order filter is proposed by Li et al. [12].

In numerical implementation, both the TV-based and the LLT model suffer from the nondifferentiability. A direct method is to replace the original nondifferentiable L^1-norm by a modified version through introducing a small regularization parameter. However, there is a trade-off between the accuracy versus the speed of convergence. A variety of efficient algorithms are developed to deal with this problem. One class is based on variable splitting and constrained optimization [3,17]. In [17], a FFT based algorithm is proposed by virtue of variable splitting and the penalty method. The split Bregman algorithm [10] and the augmented Lagrangian method [19,20], which were demonstrated to be equivalent with each other, are also efficient algorithms. Another class uses the primal-dual formulation of the original problem. Many methods of this class focus either on the primal variable or on the dual variable [4,7]. Recently, Zhu and Chan [22] presented an efficient primal-dual hybrid gradient (PDHG) algorithm, which alternates between the primal and dual variable by gradient-type methods. This algorithm is both simple and fast. Furthermore, it can be applied to a large range of restoration problems. Motivated by [22], we propose an effective and fast primal-dual based gradient algorithm to solve the fourth-order LLT model.

The following part of the paper is organized as follows. In Section 2, we recall first the LLT model in a vectorial form and then review briefly the existing methods for solving the LLT model. In the next section, we present the PDHG algorithm for solving the LLT model. In Section 4, some numerical implementation details are given. Numerical examples and comparisons are shown in Section 5. Finally, we conclude the paper in Section 6.