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A Collocation Method for Initial Value Problems of Second-Order ODEs by Using Laguerre Functions

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Abstract. We propose a collocation method for solving initial value problems of second-order ODEs by using modified Laguerre functions. This new process provides global numerical solutions. Numerical results demonstrate the efficiency of the proposed algorithm.

AMS subject classifications: 65L05, 41A30

Key words: Laguerre-Gauss collocation method using modified Laguerre functions, initial value problems of second-order ODEs.

1. Introduction

Many practical problems are governed by initial value problems of second-order ODEs. We may reformulate such problems to systems of first-order ODEs and then solve them numerically. Whereas, for saving work, it seems reasonable to solve them directly.

For notational convenience, we denote $\frac{d^r U}{dt^r}$ by $\partial_t^r U$. In many literatures, one focused on finite difference methods for the equation

$$\partial_t^2 U = f(U,t).$$

Generally, we divide those methods into two classes. In the first class of finite difference schemes, the coefficients depend on some known periods or frequencies of solutions, including exponential-fitted and trigonometrically-fitted methods, and linear multi-step method. In the second class of finite difference schemes, the coefficients are constants,

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such as Runge-Kutta-Nyström method, linear multi-step method, hybrid method, Störmer-Cowell method and prediction-correction method. Relatively, there have been less existing results on numerical methods for general equation

$$\partial_t^2 U = f(\partial_t U, U, t),$$

for which one often used Runge-Kutta-Nyström method, SDIRKN method and linear multi-step method.

Another efficient algorithm for solving initial value problems of ODEs is based on various collocations. The collocation method for first-order ODEs, could be regarded as Runge-Kutta method or linear multi-step method, see, e.g., [1, 5, 17–19, 22, 23, 27]. The collocation method has been also applied successfully to second-order ODEs, which is called as collocation-based Runge-Kutta-Nyström method in some literatures, see [16, 20, 21] and the references therein.

As is well known, spectral and pseudospectral methods employ orthogonal systems as the basis functions, and so usually provide accurate numerical results. Especially, we could use the Laguerre orthogonal approximation and interpolation to solve differential and integral equations on the half line, see [2,4,6,7,24–26,28] and the references therein. Some authors designed Legendre and Laguerre collocation methods for initial value problems of first-order ODEs, cf. [9–12]. Meanwhile, the authors investigated Legendre and Laguerre collocation methods for second-order ODEs, see [13, 29]. One of advantages of Laguerre collocation method is the ability of producing global numerical solution for all time $t \geq 0$.

There are two kinds of Laguerre collocation method. In the first class of Laguerre collocation method, one took the modified Laguerre polynomials as the basis functions. They are convergent in certain Sobolev space with the weight $e^{-\beta t}, \beta > 0$, even if the exact solutions grow very rapidly as t increases. However, it does not ensure the high accuracy in the $C(0,\infty)$-norm. In other words, even if the global weighted norm of error is small, the point-wise error might be big for large t. However, if the exact solutions are in the space $L^2(0,\infty)$, then we could use the second kind of Laguerre collocation method using the modified Laguerre functions. In this paper, we propose a new collocation method, in which we take the modified Laguerre functions as the basis functions and approximate the solutions of second-order ODEs directly.

The paper is organized as follows. The next section is for preliminary. In Section 3, we design the new numerical process, which has several merits:

- it provides the global numerical solution directly,
- it is simple to derive efficient algorithm,
- it is easy to be implemented for nonlinear problems,
- it is more natural, if the original problem is well posed in certain space without any weight.

In Section 4, we develop a multi-step version. More precisely, we first use the Laguerre-Gauss collocation method with moderate mode to obtain numerical results, and then refine them step by step. This technique simplifies computation and often leads to more accurate