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Factors Influencing the Disturbed Flow Patterns Downstream of Curved Atherosclerotic Arteries

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Abstract. Pulsatile blood flows in curved atherosclerotic arteries are studied by computer simulations. Computations are carried out with various values of physiological parameters to examine the effects of flow parameters on the disturbed flow patterns downstream of a curved artery with a stenosis at the inner wall. The numerical results indicate a strong dependence of flow pattern on the blood viscosity and inlet flow rate, while the influence of the inlet flow profile to the flow pattern in downstream is negligible.

AMS subject classifications: 76D05, 92C50, 76M10

Key words: Curved artery, atherosclerosis, blood flow, wall shear stress, flow pattern.

Dedicated to Professor Yucheng Su on the Occasion of His 80th Birthday

1. Introduction

Atherosclerosis is a disease of large- and medium-size arteries. It preferably develops and progresses at arterial branches, curved artery segments, and artery bifurcations, where the curvature could cause flow irregularity, such as flow shift, flow recirculation, secondary flow, oscillating wall shear stress and fluctuating blood pressure.

Extensive experimental and computational investigations have been made to study the blood flows in arteries with bends or bifurcations and to establish the correlation between the development of the disease and the local haemodynamics [1–11]. Friedman et al. [1] and Nerem et al. [2] have performed experiments that showed an accelerated occurrence of atherosclerosis in human subjects with a coronary geometry. Smedby [3] did an angiographic study in the femoral artery on the direction of growth of plaques, participated by 237 patients with slight or moderate atherosclerosis. Myers et al. [5] investigated the effects of flow waveform and inlet velocity profile on the flow patterns in a model based on the images of a human right coronary artery exhibiting minimal atherosclerotic lesions. Johnston et al. [7] compared models of Newtonian and Non-Newtonian flow in healthy

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right coronary arteries with no sign of atheroma. Gach et al. [8] presented a characteristic relationship between the reattachment length in the downstream of a stenosis and the stenotic Reynolds number using MRI. Talukder et al. [9] experimentally studied the effects of the number of stenosis and the distance between consecutive stenoses on the total pressure drop across a series of stenoses. Dash et al. [10] investigated the effect of catheterization on flow characteristics in a curved artery with an axisymmetric stenosis for steady flow with values of Reynolds number up to 100. Nosovitsky et al. [11] studied the flow in a coronary artery model of several degrees of stenosis-like obstruction, where the artery was narrowed around the bend from all surfaces with higher degree of stenosis. Their results showed that coronary artery curvature has an important impact on the intraluminal flow and shear stress. As a result, much has been learned about the haemodynamics in curved arteries.

Although the blood flows in curved arteries have been intensively studied in the past decades, not much work has been reported on the blood flow in single-curved arteries with severe stenosis at the inner wall. The author [12–15] has previously carried out computer simulations of blood flows in curved stenotic arteries and studied the effects of the geometry of the artery, such as the angle of the bend and the size of the stenosis, on the flow patterns. The author also examined the effect of the boundary condition at the artificial outlet boundary on the blood flow in [16] and investigated the difference in the correlations of the flow shift and the blood pressure drop to the change of the Reynolds number by varying the blood viscosity or by varying the mean inlet flow rate [17].

The aim of the present work is to investigate the dependence of the flow patterns downstream of a curved stenotic artery on the flow parameters, such as blood viscosity, inlet flow rate and inlet velocity profile. Emphasis is placed on finding how the wall shear stress and the secondary flow depend on those parameters. Numerical computations are carried out under a variety of physiological flow conditions to investigate the local flow in curved arteries with a stenosis at the inner wall of the bend.

2. Mathematical model

This study assumes that the fluid is Laminar, Newtonian, viscous and incompressible and the artery wall is rigid. These assumptions have been shown to be adequate for pulsatile blood flow simulation in artery models under physiological flow conditions by many investigators [7, 18–20]. The time dependent three dimensional Navier Stokes equations are used as the governing equations:

$$\rho(\partial \mathbf{u})/(\partial t) - \nabla \cdot [-p\mathbf{I} + \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = 0, \quad \text{in } \Omega, \tag{2.1}$$

$$\nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega, \tag{2.2}$$

where η is the viscosity of the fluid, $\mathbf{u} = (u_1, u_2, u_3)$ is the flow velocity, p is the internal pressure and ρ is the density of the fluid. The computational domain Ω is a segment of a curved artery with a stenosis at the inner wall as shown in Fig. 1(a).