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Two-Step Modulus-Based Matrix Splitting Iteration Methods for a Class of Implicit Complementarity Problems

Yan Wang[1], Jun-Feng Yin[1,*], Quan-Yu Dou[2] and Rui Li[1,3]

[1] *School of Mathematical Sciences, Tongji University, Shanghai, 200092, China*
[2] *College of Mathematics and Information Science, Zhengzhou University of Light Industry, Henan, 450002, China*
[3] *College of Mathematics Physics and Information Engineering, Jiaxing University, Zhejiang, 314001, China*

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Abstract. Modulus-based matrix splitting iteration methods were recently proposed for solving implicit complementarity problems. In this paper, to solve a class of implicit complementarity problems, two-step modulus-based matrix splitting iteration methods are presented and analyzed. The convergence theorems are established when the system matrix is an H_+-matrix. Numerical results show that the proposed methods are efficient and can accelerate the convergence performance with less iteration steps and CPU time.

AMS subject classifications: 90C33, 65F10, 65F50

Key words: Implicit complementarity problems, modulus-based matrix splitting, H_+-matrix, convergence.

1. Introduction

The complementarity problems widely arise from many applications in scientific and engineering computing, such as contact problem in elasticity, economic transportation, free boundary problems of fluid dynamics, and convex quadratic programming, inverse problems, see [1–4] for details.

Consider the following implicit complementarity problem (ICP(q,M)): Find a pair of vectors u and $r \in \mathbb{R}^n$ such that

$$u - f(u) \geq 0, \quad r := Mu + q \geq 0, \tag{1.1a}$$

$$(u - f(u))^T (Mu + q) = 0, \tag{1.1b}$$

*Corresponding author. *Email addresses:* 7ywang@tongji.edu.cn (Y. Wang), yinjf@tongji.edu.cn (J. F. Yin), 08douquanyu@tongji.edu.cn (Q. Y. Dou), lirui@tongji.edu.cn (R. Li)

where $M \in \mathbb{R}^{n\times n}$ is a given matrix, $q \in \mathbb{R}^n$ is a real vector, and $f(\cdot)$ is a mapping from $\mathbb{R}^n$ into itself. Obviously, problem (1.1) reduces to the linear complementarity problem if $f(u) = 0$.

In the past decades, a number of efficient iteration methods have been proposed for solving linear complementarity problems, for instance, projected relaxation methods [5,6], fixed-point iterations [7–9]. A class of generalized AOR (GAOR) methods was well studied in [10] and the convergence theorem was established in details. The convergence region of generalized AOR (GAOR) methods was improved by Najafi and Edalatpanah in [11] and preconditioning technique was used to solve linear complementarity problems in [12]. In particular, by combining the modulus method and the matrix splitting technique, Bai [13] presented the modulus-based matrix splitting iteration method for linear complementarity problems. Inspired by the idea, a number of variants including the two-step modulus-based matrix splitting iteration method [14], the accelerated modulus-based matrix splitting iteration method [15,16] and general modulus-based matrix splitting iteration method [17] were proposed and well studied, respectively. To suit computational requirements of the modern high-speed multiprocessor environments, the modulus-based synchronous multisplitting iteration method was presented by Bai and Zhang in [18]. For more details of multisplitting iteration methods, we refer to [19–21] and the references therein. Recently, Xia and Li [22] extended the modulus-based matrix splitting method to solve nonlinear complementarity problems, the two-step modulus-based matrix splitting iteration method [23,24] and the accelerated modulus-based matrix splitting iteration method [25, 26] were also well studied, respectively.

The implicit complementarity problem was defined in [27] and Pang [28] established a convergence theory for a certain iterative algorithm for solving the implicit complementarity problem. Moreover, variational inequality techniques [29, 30], projection operator methods [31], fix-point theory [32] were well studied for the solution of implicit complementarity problem. Motivated by the methods for linear and nonlinear complementarity problems, Hong and Li [33] first extended the modulus-based matrix splitting iteration method to solve the implicit complementarity problem, and modulus-based synchronous multisplitting iteration method was also well studied in [34]. In [35], Wang, Cao and Shi demonstrated a complete version of the convergence theory of the modulus-based matrix splitting iteration method proposed by Hong and Li [33]. In this paper, a class of two-step modulus-based matrix splitting iteration methods are proposed and studied. Further, convergence results are presented when the system matrix is an H_+-matrix. In addition, numerical experiments show that the new methods are superior to the standard modulus-based matrix splitting iteration methods for solving large sparse implicit complementarity problems.

The outline of this paper is organized as follows. Some basic notations and lemmas are presented and the two-step modulus-based matrix splitting iteration methods are proposed in Section 2, while the convergence theorems of the proposed methods when M is an H_+-matrix are established in Section 3. Numerical experiments further verify the effectiveness of the new methods in Section 4. Finally, conclusions are drawn in Section 5.