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An Implicit Scheme for Solving Unsteady Boltzmann Model Equation

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Abstract. When solving hyperbolic Boltzmann model equation with discrete velocity models (DVM), the strong discontinuity of the velocity distribution function can be captured well by utilizing the non-oscillatory and non-free parameter dissipation (NND) finite difference scheme. However, most NND scheme solvers march in time explicitly, which compromise the computation efficiency due to the limitation of stability condition, especially when solving unsteady problems. In order to improve the efficiency, an implicit scheme based on NND is presented in this paper. Linearization factors are introduced to construct the implicit scheme and to reduce the stencil size. With the help of dual time-stepping method, the convergence rate of unsteady rarefied flow simulation can be massively improved. Numerical tests of steady and unsteady supersonic flow around cylinders are computed in different flow regimes. Results are shown to prove the validity and efficiency of the implicit scheme.

AMS subject classifications: 65M10, 78A48

Key words: Rarefied gas, discrete velocity method(DVM), implicit, dual time stepping, Boltzmann model equation.

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1. Introduction

Rarefied gas flow problems involving astronautics, micro-electromechanical systems, low-pressure industry, etc., are generally studied by solving Boltzmann equation. However, due to the complexity of the collision term, it is very difficult to solve Boltzmann equation directly. Different models are introduced to replace the original collision term in Boltzmann equation, such as BGK model [2], ES-BGK model [5], Shakhov model [15],

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etc. The original collision term is greatly simplified, whereas many important characteristics of the Boltzmann equation is still preserved. There are various approaches to solving the Boltzmann equation. Apart from probabilistic numerical methods such as Direct simulation Monte Carlo (DSMC) [9], the most commonly studied methods for solving Boltzmann equation are deterministic numerical methods, such as discrete velocity models (DVM) [11, 12, 17] and spectral schemes. Fast spectral methods [18–20] has also been developed recently to solve the collision term directly, making it more accurate in solving Boltzmann equation.

Based on the discrete velocity models, researchers [9, 10, 12, 21, 23] have developed various gas kinetic theories and methods to solve Boltzmann model equations. Among which the unified gas-kinetic scheme (UGKS) [21] and discrete unified gas-kinetic scheme (DUGKS) [24] played an important role in obtaining solutions for all flow regimes. However, all the methods mentioned above solve discrete Boltzmann equations explicitly. Due to the multi-dimensional property of the velocity distribution function in Boltzmann equation, the molecular velocity need to be decoupled by DVM, resulting a large set of partial differential equations to solve. Consequently, the low convergence rate of the explicit methods due to stability condition may lead to a high computational cost, especially when solving unsteady rarefied gas problems.

To overcome the efficiency issues when solving Boltzmann equation under DVM framework, implicit schemes based on finite volume method are developed by researchers [7, 8, 13, 25] respectively. To simplify the Jacobian evaluation problem of the full implicit schemes, implicit-explicit schemes for BGK kinetic equations are developed by Pieraccini [14] et al., in which the convective term is treated explicitly, while the source term is integrated implicitly. Other methods, such as multigrid techniques [26] are also developed recently to accelerate convergence and improve efficiency of Boltzmann solvers. However, all the methods mentioned above are applied and proved efficient in solving steady Boltzmann model equations, and the more practical unsteady problems are yet to be solved.

In this paper, an implicit method based on NND finite difference scheme is developed to solve BGK-type Boltzmann model equation. Thanks to the non-oscillatory property of NND scheme, the space derivative of the flux can be discretized with a high resolution when dealing with discontinuities, making NND scheme suitable for solving supersonic Boltzmann model equation. Traditional NND scheme is constructed with a 5-point stencil, which makes it difficult to develop implicit methods. We manage to present a smaller stencil implicit scheme by introducing the linearization factors to the existing NND scheme. By means of the presented implicit scheme and the dual time stepping method, both the convergence rate and computation efficiency are massively improved when solving steady and unsteady rarefied flow problems. Numerical tests are provided in the paper to prove validity and efficiency of the scheme.

The rest of this paper is organized as follows. Section 2 provides a brief introduction to Boltzmann model equations and DVM. In Section 3, the construction of the implicit scheme is presented in detail as well as some other numerical methods for solving unsteady Boltzmann equations. The numerical tests of supersonic rarefied flow around cylinders are given in Section 4 to prove the validity of the scheme. Last section concludes the paper.