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An Improved Model Reduction Method on AIMs for N-S Equations Using Multilevel Finite Element Method and Hierarchical Basis

M. Nauman Aslam*, Jiazhong Zhang, Nannan Dang and Riaz Ahmad

School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an 710049, China

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Abstract. A numerical method is proposed to approach the Approximate Inertial Manifolds (AIMs) in unsteady incompressible Navier-Stokes equations, using multilevel finite element method with hierarchical basis functions. Following AIMS, the unknown variables, velocity and pressure in the governing equations, are divided into two components, namely low modes and high modes. Then, the couplings between low modes and high modes, which are not accounted by standard Galerkin method, are considered by AIMs, to improve the accuracy of the numerical results. Further, the multilevel finite element method with hierarchical basis functions is introduced to approach low modes and high modes in an efficient way. As an example, the flow around airfoil NACA0012 at different angles of attack has been simulated by the method presented, and the comparisons show that there is a good agreement between the present method and experimental results. In particular, the proposed method takes less computing time than the traditional method. As a conclusion, the present method is efficient in numerical analysis of fluid dynamics, especially in computing time.

AMS subject classifications: 35Q40, 76M10, 80M10, 35R01.

Key words: Fluid dynamics, model reduction, inertial manifolds, multilevel finite element method, hierarchical basis functions, nonlinear dynamics.

1. Introduction

In computational analysis, mathematical models which exhibit complex physical behavior along high fidelity and strong nonlinearity has been the focus of many researchers recently. In engineering, nonlinear continuous dynamical systems describe the vast majority of phenomena, such as complex fluid flows, fluid structure interactions. For fluid flow problems, the motion of flow exhibits complex behavior and behind this complexity is the fact that the dynamics of the systems may be the product of multiple different interacting

*Corresponding author. *Email address:* `naumanaslamgcu@gmail.com` (M. N. Aslam)

forces. It is known that some dynamic systems with continuum mechanics, are governed by a set of nonlinear partial differential equations, and some nonlinear behaviors, which can be captured by numerical solution, occur in such kind of systems. Despite the complexity of the flow topology, the entire behavior of most fluid flows is described by the so-called Navier-Stokes equations. Since in most cases, these equations do not provide the known analytical solutions, many numerical methods have been developed over the years to solve them [1-5]. For such systems, when applying discretization methods, the cost of computing time for the resulting equations is considerably expansive due to high number of degrees of freedom. Normally, finite element method is applied to approach the solution to such governing equations. Consequently, the resulting equations are mostly nonlinear dissipative evolution equation with a lot of degrees-of-freedom. In order to analyze the dynamics of the equations, the system is changed into phase-space. However, in finite dimensional phase space with higher dimension, many difficulties appear from analyzing the nonlinear dynamics both qualitatively and quantitatively. Indeed, the disadvantage of the above mentioned numerical methods is that they require considerable computing time with great difficulties, due to large number of degrees-of-freedom, and the long term behaviors of the systems have great influences from the numerical computational errors [6-7]. For such problem, there are some numerical methods. For example, the systems with local nonlinearity have been analyzed by IRS and balanced realization methods by Friswell et al. [8]. In structural dynamics with nonlinearities, there are some reduction methods mostly having numerical algorithms based on component synthesis techniques which can be efficiently used for linear dynamic systems and the solution to these problems are obtained through many numerical experiments and computational analysis [9-10]. While Slaats et al. [11] introduced a reduction method based on three modes for nonlinear dynamical systems using finite element discretization. Therefore, model reduction for high dimensional or infinite dimensional fluid dynamic systems are required to overcome such difficulties.

On the other hand, in the study of long term behaviors of dissipative nonlinear evolution systems, one encounters with the global attractors, which is compact, invariant set with finite fractal dimensions attracting all the orbits of the systems uniformly, such attractors have complicated and dynamic structures [12]. A theoretical approach was shown that there is a approximate inertial manifold for the long term behavior of some dissipative partial differential equations in Titi [13]. Consequently, it has been proved that infinite-dimensional dissipative systems can be reduced to finite dimensional systems by reduction technique. Thus, a number of methods have been applied to construct a finite system exhibiting asymptotic dynamic behavior in the original dynamic system [14-15]. More, an important feature of nonlinear dynamics is given related to model reduction, which explains the asymptotic properties based on spectral theory and decomposition process of the dynamical systems [16]. Therefore, developing a feasible model reduction method is very urgent as continuum dynamic systems are studied numerically.

For decades, the concept of inertial manifold is an important development in the study of systems with complicated attractor, since it reduces an infinite dimensional problem into a finite dimensional one without introducing errors [17]. Infinite dimensional dynamics