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On Further Study of Bivariate Polynomial Interpolation over Ortho-Triples

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Abstract. In this paper, based on the recursive algorithm of the non-tensor-product-typed bivariate divided differences, the bivariate polynomial interpolation is reviewed firstly. And several numerical examples show that the bivariate polynomials change as the order of the ortho-triples, although the interpolating node collection is invariant. Moreover, the error estimation of the bivariate interpolation is derived in several cases of special distributions of the interpolating nodes. Meanwhile, the high order bivariate divided differences are represented as the values of high order partial derivatives. Furthermore, the operation count approximates $\mathcal{O}(n^2)$ in the computation of the interpolating polynomials presented, including the operations of addition/substractions, multiplication, and divisions, while the operation count approximates $\mathcal{O}(n^3)$ based on radial basis functions for sufficiently large n.

AMS subject classifications: 65D05, 41A20

Key words: Bivariate polynomial interpolation, non-tensor product type, error estimation, ortho-triple, radial basis function.

1. Introduction

There are several reasons for studying multivariate approximation theory and methods, which ranges from a need to represent functions in computer calculations to an interest in the mathematics of the subject. Specifically, as we know, objectives fixed at the triples are more stable than at points separately in structure mechanics, so problems of bivariate interpolation from the information collected over the triples need to be solved. By means of a new non-tensor-product-typed divided difference algorithm for two variable, Salzer [24] proposed an interpolating polynomial scheme over several ortho-triples.

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He examined the existing interpolation formulas for more than one dimension, and found that the desirability of at least three properties should be possessed by any basic scheme, i.e., truly irregular distributions of points, fairly simple recursive scheme for calculating successive interpolating coefficients, and limiting confluent forms.

Obviously, bivariate polynomial interpolation over ortho-triples is part of bivariate polynomial interpolation in the form of non-tensor product type [2, 4, 7]. As all know, non-tensor-product-typed bivariate polynomial interpolation has been studied extensively for many decades, which is important but complicated due to not arising in the simple univariate case directly [8]. In fact, the geometric distribution of interpolating nodes is crucial for determining the solvability of the interpolation problem [29, 33]. Thus the subject on non-tensor-product-typed bivariate interpolation has been challenging and attracted many people's attention, which belongs to scattered data approximation. Excellent surveys on recent accomplishments can be found in scattered data interpolation, which is based on radial basis functions [3,34]. The radial basis function method has been one of the most often applied methods in multivariate approximation theory when the task is to approximate scattered data in several dimension. Besides these, multivariate Birkhoff interpolation has been investigated [1, 5]. Classic Bezout Theorem has played a vital pole on constructing bivariate interpolating polynomials over equally partitioned triangular grids [29]. What's more, Much attention has been paid on the study of convex preserving scattered data interpolation [6,14,15,17], which is based upon classic multivariate splines [33]. Zhan [35] proposed that there existed a C^3 piecewise polynomial of degree 7 interpolating arbitrarily given values and derivatives of orders up to 3. Zhou and Lai [36] constructed several new schemes to improve the order of approximation, which is a mixture of B-forms of polynomials. Lai [9] used bivariate C^1 cubic splines to deal with convexity preserving scattered data interpolation problem. By means of interpolation along a piecewise algebraic curve, Zhu and Wang [37] established Lagrange interpolation sets for bivariate spline spaces over cross-cut partitions. The above construction of the bivariate spline interpolation needs spline bases in the minimal supports [16, 18–20, 30, 33] based on the theory of piecewise surfaces [10–13] and the Conformality of Smoothing Cofactor Method [33].

To be mentioned, non-tensor-product-typed bivariate interpolation needs deep and constant attention. Classic tensor-product-typed bivariate interpolation methods such as continued fraction interpolation [26–28] need be considered and applied in the case of non-tensor-product type. Hence, based on a new symmetrical algorithm of partial inverse differences in $\mathbf{R}^2$, a bivariate continued fraction interpolating scheme has been constructed over ortho-triples [32]. Moreover, three-term recurrence relations for branched continued fractions have been determined, and the modified branched continued fractions interpolation over pyramid-typed grids in $\mathbf{R}^3$ has been constructed with the algorithm of partial inverse differences in tensor-product-like manner in [31]. Furthermore, base on recursive algorithms of the non-tensor-product-typed bivariate divided differences and partial inverse differences, bivariate polynomial [22, 23] and continued fraction [21] interpolating schemes are established, respectively. Meanwhile, Further study of bivariate polynomial interpolation over ortho-triples shall be made in the paper.

A brief outline of this paper is organized as the following. Firstly, in Section 2, we