A Meshless and Parallelizable Method for
Differential Equations with Time-Delay

Shulin Wu¹,* and Chengming Huang²,³

¹ School of Mathematics and Statistics, Sichuan University of Science and Engineering, Zigong 643000, China
² School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China
³ Hubei Key Laboratory of Engineering Modeling and Scientific Computing, Huazhong University of Science and Technology, Wuhan 430074, China

Received 7 September 2016; Accepted (in revised version) 24 March 2017

Abstract. Numerical computation plays an important role in the study of differential equations with time-delay, because a simple and explicit analytic solution is usually unavailable. Time-stepping methods based on discretizing the temporal derivative with some step-size $\Delta t$ are the main tools for this task. To get accurate numerical solutions, in many cases it is necessary to require $\Delta t < \tau$ and this will be a rather unwelcome restriction when $\tau$, the quantity of time-delay, is small. In this paper, we propose a method for a class of time-delay problems, which is completely meshless. The idea lies in representing the solution by its Laplace inverse transform along a carefully designed contour in the complex plane and then approximating the contour integral by the Filon-Clenshaw-Curtis (FCC) quadrature in a few fast growing subintervals. The computations of the solution for all time points of interest are naturally parallelizable and for each time point the implementations of the FCC quadrature in all subintervals are also parallelizable. For each time point and each subinterval, the FCC quadrature can be implemented by fast Fourier transform. Numerical results are given to check the efficiency of the proposed method.

AMS subject classifications: 65M15, 65D05, 65D30

Key words: Delay differential equations, meshless/parallel computation, contour integral, Filon-Clenshaw-Curtis quadrature.

1. Introduction

Delay differential equations (DDEs) arise from various applications, like biology [27], control of dynamic systems [5, 21], circuit engineering [32] and many others. A DDE differs from an ordinary differential equation (ODE) in that it depends not only on the
Figure 1: For DDE (1.1) with $\tau = 0.05$, the approximation of the numerical solutions generated by the implicit Euler method (left) and the trapezoidal rule (right) to the exact solution, the cosine function $u(t) = \cos\left(\frac{\pi}{2\tau}t\right)$.