

## Learning Non-Negativity Constrained Variation for Image Denoising and Deblurring

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**Abstract.** This paper presents a heuristic Learning-based Non-Negativity Constrained Variation (L-NNCV) aiming to search the coefficients of variational model automatically and make the variation adapt different images and problems by supervised-learning strategy. The model includes two terms: a problem-based term that is derived from the prior knowledge, and an image-driven regularization which is learned by some training samples. The model can be solved by classical  $\varepsilon$ -constraint method. Experimental results show that: the experimental effectiveness of each term in the regularization accords with the corresponding theoretical proof; the proposed method outperforms other PDE-based methods on image denoising and deblurring.

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**Key words:** Learning idea, TV-based model, constraint,  $\varepsilon$ -constraint method, image restoration.

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### 1. Introduction

Variational methods have been widely applied to various areas of image restoration and remain active in mathematical research of image processing. One of the most remarkable work is total variation (TV) which was first introduced into image denoising in the seminal work [36] by Rudin, Osher and Fatemi. The mostly used version of discrete TV [33, 38] is given by

$$\|u\|_{TV} = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \sqrt{(u_{i+1,j} - u_{i,j})^2 + (u_{i,j+1} - u_{i,j})^2}, \quad (1.1)$$

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which is the discretization of  $\int_{\Omega} |\nabla u| dx$  by pixel. The unconstrained version of TV-based model reads

$$\min_u \frac{1}{2} \|Ku - f\|^2 + \beta \|u\|_{TV}, \quad (1.2)$$

where  $f : \Omega \rightarrow \mathbb{R}$  is the degraded image,  $u : \Omega \rightarrow \mathbb{R}$  is the recovered image,  $K$  is the blurring operator, and the coefficient  $\beta$  usually depends on the noise level. The first term of (1.2) is the  $l_2$  norm of  $Ku - f$  which guarantees inheritance from the observed image  $f$ , and the second term is total variation which helps to remove the noise. This TV-based deblurring model solves different problems for different choices of  $K$ , for example,  $K = I$  for image denoising. In this paper, we assume that  $K$  is derived from a point spread function (PSF), namely,  $K$  is known. Owing to excellent effectiveness of total variation on reducing noise and preserving edges, TV-based models have ignited plenty of research in dealing with image denoising and deblurring [12, 19, 21, 41, 46, 47], image inpainting [6, 10, 14, 15], image superresolution [13], image segmentation [11, 16, 26, 28, 32, 40] and other image processing problems [8].

Adding the non-negativity constraint to these TV-based models helps to recover the image when the constraint is physically meaningful. For example, images whose pixels represent the number of photon pairs must be non-negative. In contrast to clipping the solution to TV-based model, enforcing the constraint in the model can achieve better performance [17, 18]. Besides, the non-negative output images are the requirement of some applications such as the medical imaging [20, 34], gamma ray spectral analysis [31, 37] and so on. Krishnan et al. [25, 26] focused on the TV-based deblurring model with non-negativity constraint, called NNCGM, and solved it with the primal-dual active-set strategy [9, 24]. The interested reader is referred to [3, 22, 44] for other constrained algorithms.

All the above TV-based models are manually designed with the insight of individual problems, which sometimes restricts the strength of these TV-based models. Recently, a learning-based PDEs (L-PDE) model was proposed by Liu et al. in their seminal work [29]. Based on the proficiency of diffusion, they combined the differential operators with some image-driven coefficients to make PDEs adaptive to different problems. The model was proven to be successful in image denoising, edge detection and image segmentation [29].

Inspired by L-PDE methods, we propose a learning-based non-negativity constrained variation which extends the traditional TV-based models. The proposed variation contains two terms: a problem-based term that is derived from the prior knowledge, and an image-driven regularization which is learned by some training samples so as to make the variation adaptive to different images as well as problems. With the help of supervised-learning strategy, the optimal coefficients of the variation are searched automatically.

The rest of the paper is organized as follows. In Section 2, we propose a heuristic non-negativity constrained variation (NNCV) model, and then we give a learning-based form of our proposed model (L-NNCV) and solve it with  $\varepsilon$ -constraint method. Section 3 presents experimental results and Section 4 concludes this paper. Appendices A-B give supplementary materials containing extended proofs and mathematical derivations.