A Vertex-Centered and Positivity-Preserving Finite Volume Scheme for Two-Dimensional Three-Temperature Radiation Diffusion Equations on General Polygonal Meshes

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Abstract. Two-dimensional three-temperature (2-D 3-T) radiation diffusion equations are widely used to approximately describe the evolution of radiation energy within a multimaterial system and explain the exchange of energy among electrons, ions and photons. In this paper, we suggest a new positivity-preserving finite volume scheme for 2-D 3-T radiation diffusion equations on general polygonal meshes. The vertex unknowns are treated as primary ones for which the finite volume equations are constructed. The edge-midpoint and cell-centered unknowns are used as auxiliary ones and interpolated by the primary unknowns, which makes the final scheme a pure vertex-centered one. By comparison, most existing positivity-preserving finite volume schemes are cell-centered and based on the convex decomposition of the co-normal. Here, the co-normal decomposition is not convex in general, leading to a fixed stencil of the flux approximation and avoiding a certain search algorithm on complex grids. Moreover, the new scheme effectively alleviates the numerical heat-barrier issue suffered by most existing cell-centered or hybrid schemes in solving strongly nonlinear radiation diffusion equations. Numerical experiments demonstrate the second-order accuracy and the positivity of the solution on various distorted grids. For the problem without analytic solution, the contours of the numerical solutions obtained by our scheme on distorted meshes accord with those on smooth quadrilateral meshes.

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1. Introduction

Radiation diffusion equations arise in a wide range of applications such as radiation hydrodynamics and astrophysical plasmas. If the radiation field is not in thermodynamic equilibrium with the material, a system of coupled time-dependent diffusion equations, which is called non-equilibrium radiation diffusion equations, is used to describe the energy transport. Due to its strong nonlinear phenomena, strong discontinuous interfaces and tight coupling, the numerical simulation of such equations is very challenging and has drawn many researchers' attention.

In recent years, some efficient numerical methods were suggested for solving the non-equilibrium two dimensional two-temperature (2-D 2-T) radiation diffusion equations [5, 15–17, 28–30, 32, 46, 48]. Most of them focused on high-order time integrations and the nonlinear iteration technique on rectangular meshes. Meanwhile, some monotone schemes were also studied. In [51], the authors designed two finite volume element schemes and proved that one of them is monotonic under some geometric conditions and another is monotonic under some repairing techniques. In [12], a monotone tailored finite point method was suggested for solving the non-equilibrium radiation diffusion equations. Recently, a moving mesh finite difference method was proposed in [44].

In the simulation of laser-driven implosion of a fuel capsule in inertial confinement fusion experiments, a finer and more complicated model, involving the two-dimensional three-temperature (2-D 3-T) radiation diffusion equations, is widely used to describe the evolution and exchange of energy among electrons, ions and photons [6, 11, 14, 18, 27]. Since 2000s, some numerical methods have been developed for 2-D 3-T radiation equations, such as finite volume method and finite element method. In [27], a fully implicit finite volume scheme combined with parallel adaptive multigrid method was suggested in the framework of UG. In [14], the authors designed a symmetric finite volume element (SFVE) method with a preconditioning technique, a mesh adaptation algorithm and a two-grid procedure. A two-level iterative method was proposed in [43] for the numerical simulation of 2-D 3-T radiation diffusion equations, based on the Jacobian-free Newton-Krylov (JFNK) framework for preconditioning. In [47], two substructuring nonoverlapping domain decomposition preconditioners were employed to solve the SFVE discretization of 2-D 3-T radiation diffusion equations with strongly discontinuous coefficients. In [7], two finite volume element schemes were constructed on triangular meshes. The authors in [31] adopted the freezing coefficient method to linearize the nonlinear equations and then solved the resulting equations by Raviart-Thomas mixed finite element method.

Unfortunately, numerical methods mentioned above for 2-D 3-T radiation diffusion equations were discussed only on triangular or quadrilateral meshes. In many applications such as radiation hydrodynamics (RHD), the meshes are typically distorted, concave or have hanging nodes due to the complex fluid flow. Hence, solving 3-T radiation diffusion equations on distorted polygonal meshes is very interesting and important. In [45], a Lions domain decomposition algorithm based on a cell functional