Numer. Math. Theor. Meth. Appl. Vol. **12**, No. 4, pp. 1191-1212
doi: 10.4208/nmtma.OA-2018-0091 November 2019

Notes on New Error Bounds for Linear Complementarity Problems of Nekrasov Matrices, B-Nekrasov Matrices and QN-Matrices

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Received 15 February 2018; Accepted (in revised version) 29 November 2018

Abstract. In this paper, we give new error bounds for linear complementarity problems when the matrices involved are Nekrasov matrices, B-Nekrasov matrices and QN-matrices, respectively. It is proved that the obtained bounds are better than those of Li et al. (New error bounds for linear complementarity problems of Nekrasov matrices and B-Nekrasov matrices, Numer. Algor., 74 (2017), pp. 997–1009) and Gao et al. (New error bounds for linear complementarity problems of QN-matrices, Numer. Algor., 77 (2018), pp. 229–242) in some cases, respectively.

AMS subject classifications: 65G50, 90C31, 90C33

Key words: Error bounds, linear complementarity problem, Nekrasov matrices, B-Nekrasov matrices, QN-matrices.

1. Introduction

The linear complementarity problem is to find a vector $x \in \mathbb{R}^n$ such that

$$x \geq 0, \quad Ax + q \geq 0, \quad (Ax+q)^T x = 0, \tag{1.1}$$

or to show that no such vector x exists, where $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$. We abbreviate this problem by LCP(A, q). Many problems can be posed in the form (1.1). For instance, problems in linear and quadratic programming, the problem of finding a Nash equilibrium point of a bimatrix game or some free boundary problems of fluid mechanics (see [1-3,29-32]). It is well known that a real square matrix A is called

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a P-matrix if all its principal minors are positive. A is a P-matrix if and only if the LCP(A, q) has a unique solution x^* for any $q \in R^n$ (see [2]).

Some error bounds for LCPs of P-matrices are derived (see [4-7]). Particularly, when the matrix A of the LCP(A, q) (1.1) is a P-matrix, Chen and Xiang [4] derived the following error bound

$$\| x - x^* \|_\infty \leq \max_{d \in [0,1]^n} ||(I - D + DA)^{-1}||_\infty \| r(x) \|_\infty,$$

where x^* is the solution of LCP(A, q), $r(x) := \min(x, Ax + q), D = \mathrm{diag}(d_i)$ with $0 \leq d_i \leq 1$, and the min operator denotes the componentwise minimum of two vectors.

When the involved matrix belongs to a subclass of P-matrices, such as H-matrices with positive diagonals, B-matrices, DB-matrices, SB-matrices, MB-matrices, B^S-matrices and weakly chained diagonally dominant B-matrices, many error bounds for the LCPs (1.1) are achieved in the literature (see [8-17, 21-24, 27-28]). In [15,16], error bounds for linear complementarity problem with Nekrasov matrices and B-Nekrasov matrices are presented respectively. Recently, Li et al. [21] provided new error bounds for LCPs(A, q) associated with Nekrasov matrices and B-Nekrasov matrices and Gao et al. [23] presented a new error bound for LCP(A, q) involved with a QN-matrix, which are only dependent on the entries of the matrix A.

In this paper, we find new error bounds of linear complementarity problem when the involved matrices are Nekrasov matrices, B-Nekrasov matrices and and QN-matrices. It is proved that the given bounds improve corresponding bounds of [21], for Nekrasov matrices and B-Nekrasov matrices, and [23], for QN-matrice in some cases. In particular, when the positive diagonal entries of the related matrix A are located in an interval $(0, 1]$, it is proved that the new bound is generally sharper than that of Remark 2.4 in [9]. Related numerical examples show that the new bounds are tighter than those derived recently.

2. A new error bound for LCPs of Nekrasov matrices

Let us first introduce some basic notations and some classes of matrices. We denote $N := \{1, \cdots, n\}$ and by $e := (1, \cdots, 1)^T$ the unit column vector of n elements. A Z-matrix is a matrix whose off-diagonal elements are nonpositive and a nonsingular M-matrix is a Z-matrix with nonnegative inverse. Given a real matrix A, its comparison matrix $\langle A \rangle = (\tilde{a}_{ij}) \in \mathbb{R}^{n \times n}$ defined by setting $\tilde{a}_{ii} = |a_{ii}|$ and $\tilde{a}_{ij} = -|a_{ij}|$, $i \neq j,\ i, j \in N$. If $\langle A \rangle$ is an M-matrix, then A is called an H-matrix. we say that a matrix $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ is strictly diagonally dominant by rows (SDD) if $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$ for all $i, j \in N$.

In what follows, we recall the definition of Nekrasov matrices [18-19] and prepare several fundamental lemmas.

Definition 2.1. *Let $A = (a_{ij}) \in \mathbb{R}^{n \times n}, n \geq 2$, with $a_{ii} \neq 0$, $i \in N$. We say that A is a Nekrasov matrix if, for all $i \in N$,*

$$|a_{ii}| > h_i(A),$$