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Electromagnetic Scattering by a Chiral Grating in a Homogeneous Chiral Environment and its Finite Element Method with Perfectly Matched Absorbing Layers

Deyue Zhang[1,2,*] and Fuming Ma[1]

[1] *School of Mathematics, Jilin University, Changchun 130012, China.*
[2] *School of Computer Science and Technology, Jilin University, Changchun 130012, China.*

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Abstract. The scattering of time-harmonic electromagnetic waves propagating in a homogeneous chiral environment by a chiral grating is studied. The problem is simplified to a two-dimensional scattering problem, and the existence and the uniqueness of solutions are discussed by a variational approach. The diffraction problem is solved by a finite element method with perfectly matched absorbing layers. Our computational experiments indicate that the method is efficient.

Key words: Chiral media; chirality admittance; Maxwell equations; perfectly matched layer.

AMS subject classifications: 35Q60, 78A45

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1 Introduction

The phenomenon of optical activity in special materials has been known since the beginning of the last century. Whereas optical activity has been considered in optics and in quantum mechanics for many years, its analysis within the framework of the classical electromagnetic field theory is much more recent. Recently, there has been a considerable interest in the study of scattering and diffraction by chiral media. In general, the electromagnetic fields inside the chiral medium are governed by Maxwell equations together with the Drude-Born-Fedorov equations in which the electric and magnetic fields are coupled. The chiral media is characterized by the electric permittivity ε, the magnetic permeability μ and the chirality measure β. On the other hand, periodic structures (gratings) have received increasing attentions through the years because of important applications in integrated optics, optical lenses, antireflective structures, et al.

Scattering by an obstacle in a chiral medium—although somewhat exotic at a first glance—constitutes an attractive problem. Some illustrative examples are the turbid chiral media [12], the Bruggeman homogenization of chiral composites [9], the method of moments for scattering

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*Correspondence to: Deyue Zhang, School of Mathematics, Jilin University, Changchun 130012, China/School of Computer Science and Technology, Jilin University, Changchun 130012, China. Email: zhangdeyue@126.com

by a chiral obstacle in a chiral environment [12]; the solvability of the latter scattering problem is studied in [2].

Recently, we studied the scattering by a chiral grating in an achiral medium in [14]. We also solved the diffraction problem by a finite element method with perfectly matched absorbing layers in [15]. In this paper, we consider the scattering of time-harmonic electromagnetic waves propagating in a homogeneous chiral environment by a chiral grating. The structure is periodic in x_1-direction and invariant in x_3-direction. We simplify the scattering problem to a two-dimensional one, and discuss the well-posedness of the scattering problem by a variational approach. We solve the diffraction problem by a finite element method with perfectly matched absorbing layers. We use the PML technique to truncate the unbounded domain to a bounded one which attenuates the outgoing waves in the PML region. Some numerical experiments are also carried out to illustrate the advantages of our method. An important step here is to derive the boundary conditions and the PML equations in the layers. This is done by using the Bohren decomposition of the electromagnetic fields. We emphasize that the variational method is very general.

For some interesting explanation and references of the model equations, we refer to Lakhtakia [10] and Lakhtakia, Varadan, Varadan [11] (non-periodic chiral structures), Ammari and Nédélec [1]. Results and references on closely related periodic structures may be found in Bao and Dobson [4], Chen and Friedman [5], Dobson and Friedman [7] and Gerlach [8].

The outline of this paper is as follows. In Section 2, the Maxwell equations and the constitutive equations, the Drude-Born-Fedorov equations, and the scattering problem are presented. We reformulate the problem in Section 3. The existence and the uniqueness of solutions are discussed by a variational approach. The energy distribution for the diffraction problem is studied in Section 4. In Section 5, we introduce our PML formulation, and establish the existence, uniqueness and convergence of the PML formulation. Finally, in Section 6, we present several numerical examples to illustrate the advantages of our method.

2 The scattering problem

Let us consider the propagation of time-harmonic electromagnetic waves. The electromagnetic fields are governed by the time-harmonic (time dependence $e^{-i\omega t}$) Maxwell's equations

$$\nabla \times \mathbf{E} - i\omega \mathbf{B} = 0, \tag{1}$$

$$\nabla \times \mathbf{H} + i\omega \mathbf{D} = 0, \tag{2}$$

where $\mathbf{E}, \mathbf{H}, \mathbf{D}$ and $\mathbf{B}$ denote the electric field, the magnetic field, the electric and magnetic displacement vectors in $\mathbb{R}^3$, respectively. For chiral media, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$ and $\mathbf{B}$ satisfy the Drude-Born-Fedorov constitutive equations:

$$\mathbf{D} = \varepsilon(x)(\mathbf{E} + \beta(x)\nabla \times \mathbf{E}), \tag{3}$$

$$\mathbf{B} = \mu(x)(\mathbf{H} + \beta(x)\nabla \times \mathbf{H}), \tag{4}$$

where $x = (x_1, x_2, x_3)$, ε is the electric permittivity, μ is the magnetic permeability, and β is the chirality admittance. After putting equations (1)-(4) together, the Maxwell's equations may be written as

$$\nabla \times \mathbf{E} = (\gamma(x))^2\beta(x)\mathbf{E} + i\omega\mu\left(\frac{\gamma(x)}{k(x)}\right)^2\mathbf{H}, \tag{5}$$

$$\nabla \times \mathbf{H} = (\gamma(x))^2\beta(x)\mathbf{H} - i\omega\varepsilon\left(\frac{\gamma(x)}{k(x)}\right)^2\mathbf{E}, \tag{6}$$