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A CONDITIONAL STABILITY FOR AN INVERSE PROBLEM ARISING IN GROUNDWATER POLLUTION*

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Abstract *An inverse problem of determining magnitude of groundwater pollution in a hydrologic region is investigated. By applying integral identity methods, a conditional stability for the inverse problem here is constructed with aids of an optimal adjoint problem and a suitable topology.*

Key words *Groundwater pollution; inverse problem of determination of source term; integral identity method; conditional stability.*

AMS(2000)subject classifications 35R30, 76S05

1 Introduction

Groundwater is an important drinkable water resource, and its pollution has become more and more serious with the development of economics and society. Information of the characteristics of specific groundwater pollution sources is necessary for the protection and management of polluted aquifers. But many times the source characteristics (location, magnitude, and duration of activity) are not known ([1-2],for instance). One of the effective ways to deal with this kind of problems is to make use of mathematical methods, especially by inverse-problem method ([3], for instance).

In fact, since the data can not be measured by direct ways in many cases, some additional information is needed to decide the unknown sources and the aquifer parameters according to the models, which always leads to inverse problems.

The problem we are concerned is groundwater vitriol pollution in Fengshui, Zibo, China,

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which is a relatively integrated unit of hydrologic geology. In this region, Yuedian and Zhanghua Wellspring were established in 20th 80'. But with the excess exploitation of mines, vitriol pollution in the groundwater has became more and more serious. Our aim is to try to work out an average magnitude of the pollution sources with inverse-problem method based on transportation model of the vitriol solute and additional observations of concentration at observed sites. In this paper, we will only investigate the stability for the inversion of the source magnitude. As for numerical computation, we will discuss and carry out it in other occasions.

As we know, there are lots of researches on uniqueness of inverse problems of source terms, but for stability analysis, there seems fewer studies in the literatures we have. Our methodology is based on an integral identity relation between changes of known data and corresponding varies of unknown functions. It seems an innovation to discuss stability of inverse problems with the method used in this paper.

Now let us consider the corresponding forward problem. Just as we know, under some suitable hypothesis for the actual hydrologic conditions, the vitriol transportation here can be characterized using the following 1-D advection-dispersion equation ([3], for instance):

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} - a_L u \frac{\partial^2 c}{\partial x^2} + \lambda c = q, \ (x,t) \in D_T. \tag{1}$$

where $D_T = (0,l) \times (0,T)$, and q: average concentration of pollutants seeping into the aquifer per unit time, which represents the magnitude of pollution sources.

On actual conditions, suppose that the seeping strength is only concerned with space variable. That is to assume $q = q(x)$. According to the known measured data, initial boundary value conditions are given below:

$$\begin{aligned} &c(0,t) = g_1(t), \quad 0 \le t \le T, \\ &c(l,t) = g_2(t), \quad 0 \le t \le T, \\ &c(x,0) = c_0(x), 0 \le x \le l. \end{aligned} \tag{2}$$

Hence, by model (1), if the aquifer parameters and the seeping magnitude are all known, we can work out concentration distribution $c = c(x,t)$ from the given initial boundary value condition (2).

In this paper, our problem is that the source magnitude function $q = q(x)$ is unknown, which needs to be decided by some additional data. According to the actual cases, the additional data are the final observations at T given as follows:

$$c(x,T) = c_T(x), \ 0 \le x \le l. \tag{3}$$