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INVERSE SCATTERING PROBLEMS BY SINGULAR SOURCE METHODS*

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Abstract *The inverse scattering problems are to detect the property of obstacles from the measurements outside the obstacles. One of important research areas in this topic is the recovery of boundary property for impenetrable obstacles. In this paper, we would like to give a brief review about the recently developed singular source methods. There are three different methods in this category, namely, linear sampling method, point-source method and probe method. We also present some recent new results about the probe method.*

Key words *Inverse scattering, Helmholtz equation, singular source, superposition, probe method.*

AMS(2000)subject classifications 35R30, 35J05

1 Introduction

The inverse scattering problem is to detect the properties of a scatterer from the information measured outside the scatterer. The general configuration of inverse scattering is as follows. If we send some incident wave such as acoustic wave or electromagnetic wave to an obstacle, then this incident wave will be scattered by the obstacle and produce the so-called scattered wave outside the obstacle. The inverse scattering problem aims to obtain some properties (position, shape or interior structure) of obstacle from the information contained in the scattered wave.

From the point of view of obstacle, we can classify the scattering problems in terms of the property of obstacle as two classes, namely, impenetrable obstacle scattering and penetrable obstacle scattering. For the impenetrable obstacle, the scattering is determined completely by the boundary of obstacle while the interior property of the obstacle has nothing to do with

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the scattering process. However, for penetrable obstacle, both the boundary property and the interior structure of the obstacle determine the scattering. In this case, the scattering process is determined by the wave inside and outside the obstacle, which is connected by some transmission condition on the boundary. Therefore, the obstacle should be penetrable if we want to determine its interior property. It is well known that the scattering of penetrable obstacle is more complicated, compared with the impenetrable obstacle scattering.

If we consider the frequency of incident wave, then the scattering problems can be divided as high frequency scattering and scattering for incident wave with wave number in the resonance domain. In former case, some linearization techniques such as geometry optical approximation can be used to obtain a linear inverse scattering problem ([1,19]). However, in the later case where the wavelength is comparable to the size of the obstacle, we should consider the nonlinear inverse problems generally ([4]).

For inverse scattering problems of impenetrable obstacles, some well-known inversion schemes such as optimization methods have been developed early ([8,9,10]). Roughly speaking, these methods consider the obstacle boundary as the solution of some nonlinear equation from the boundary space to the near field data space determined from the boundary condition, and then try to find the approximate general solution of this equation in some sense. These methods concentrate on the approximate solution of boundary shape without the consideration of the uniqueness. In recent years, a new technique called as singular source method for the inversion of obstacle boundary has been developed theoretically and numerically. This method is theoretically exact in the sense that it determines the boundary by the blowing-up property of some indicator function. It is also implementable numerically by concrete construction of indicator function for different kinds of problems. This paper gives an overview on inversion theory in this category, and pay special attentions to one of these methods, probe method.

2 Acoustic wave inverse scattering models

Consider acoustic wave propagation in inhomogeneous media in $\mathcal{R}^m, m=2,3$. Denote by $U(x,t)$ the velocity potential. Then it satisfies

$$\frac{\partial^2 U}{\partial t^2}+\gamma(x)\frac{\partial U}{\partial t}-c^2(x)\Delta U=0, \tag{1}$$

where $\gamma(x)$ is the dissipative coefficient, $c(x)$ the wave speed. Let $U(x,t)=u(x)e^{-i\omega t}$ with frequency $\omega>0$, then $u(x)$ satisfies

$$\Delta u+k^2 n(x)u=0, \quad x\in\mathcal{R}^m, \tag{2}$$

where the wave number $k=\dfrac{\omega}{c_0}$, $n(x)=\dfrac{c_0^2}{c^2(x)}+i\dfrac{1}{\omega}\dfrac{c_0^2}{c^2(x)}\gamma(x)$. The function $n(x)$, called as