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# UPDATING AND DOWNDATING FOR PARAMETER ESTIMATION WITH BOUNDED UNCERTAIN DATA *

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**Abstract** *The bounded parameter estimation problem and its solution lead to more meaningful results. Its superior performance is due to the fact that the new method guarantees that the effect of the uncertainties will never be unnecessarily overestimated. We then consider how to update and downdate the bounded parameter estimation problem. When updating and downdating of* SVD *are used to the new problem, special technologies are taken to avoid forming $U$ and $V$ explicitly, then increase the algorithm performance. Because of the link between the bounded parameter estimation and Tikhonov regularization procedure, we point out that our algorithms can also be used to modify regularization problem.*

**Key words** *Parameter estimation,* SVD*, Updating and downdating*

**AMS(2000)subject classifications** $65F05, 65F10, 65F35$

## 1 Introduction

The important problem in estimation is to recover, to good accuracy, a set of unobservable parameters from corrupted data. Among the most notable variations of the linear least-squares criterion([1][4]) is the parameter estimation problem with priori bounds on the size of the allowable corrections to the data. The most useful scenarios is that the priori bounds on the uncertain data are known [7]. The solution leads to more meaningful results in the sense that it guarantees that the effect of the uncertainties will never be unnecessarily overestimated, beyond what is reasonably assumed by the priori bounds.

Let $A \in R^{m\times n}$ be a given matrix with $m \geq n$ and $b \in R^m$ a given vector, both of which are assumed to be linearly related via an unknown vector of parameter $x \in R^n$,

$$b = Ax + v. \tag{1.1}$$

The vector $v \in R^m$ denotes measurement noise and it explains the mismatch between $Ax$ and

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the given vector (or observation) $b$. We assume that the "true" coeffient matrix is $A+\delta A$ and that we only know an upper bound on the perturbation $\delta A$,

$$\|\delta A\|_2 \leq \eta, \tag{1.2}$$

with $\eta$ being known. Likewise, we assume that the "true" observation vector is $b+\delta b$ and that we know an upper bound $\eta_b$ on the perturbation $\delta b$,

$$\|\delta b\|_2 \leq \eta_b. \tag{1.3}$$

We then pose the problem of finding an estimate that performs "well" for any allowed perturbation $(\delta A, \delta b)$. More specifically, we pose the following min-max problem (see problem 1 in [7]).

**Original problem** Given $A \in R^{m\times n}$, with $m \geq n, b \in R^m$, and nonnegative real numbers $(\eta, \eta_b)$, determine, if possible, an $\hat{x}$ that solves

$$\min_{\hat{x}} \max\{\|(A+\delta A)\hat{x} - (b+\delta b)\|_2 : \|\delta A\|_2 \leq \eta, \|\delta b\|_2 \leq \eta_b\}. \tag{1.4}$$

We note that if $\eta = 0 = \eta_b$, then problem (1.4) reduces to a standard least-squares problem. Therefore we shall assume throughout that $\eta > 0$. We can see from [7] that the solution to the above min-max problem is independent of $\eta_b$.

In this paper, we first consider how to update the original problem, that is:

**Problem 1** Having solved the original problem, how can we make use of the information produced during the procedure of solving the original problem to compute the updating parameter estimation problem:

$$\min_{\hat{x}} \max\{\|(\tilde{A}+\delta \tilde{A})\hat{x} - (\tilde{b}+\delta \tilde{b})\|_2 : \|\delta \tilde{A}\|_2 \leq \eta, \|\delta \tilde{b}\|_2 \leq \eta_b\}, \tag{1.5}$$

where $\tilde{A} = \begin{bmatrix} A \\ a^T \end{bmatrix}$, $\tilde{b} = \begin{bmatrix} b \\ b_{m+1} \end{bmatrix}$, $(\eta, \eta_b)$ are the same as those in original problem.

Similarly, we formulate the downdating parameter estimation problem as:

**Problem 2** Having solved the original problem, how can we make use of the information produced during the procedure of solving the original problem to compute the downdating parameter estimation problem:

$$\min_{\hat{x}} \max\{\|(\tilde{A}+\delta \tilde{A})\hat{x} - (\tilde{b}+\delta \tilde{b})\|_2 : \|\delta \tilde{A}\|_2 \leq \eta, \|\delta \tilde{b}\|_2 \leq \eta_b\}, \tag{1.6}$$

where $A = \begin{bmatrix} \tilde{A} \\ a^T \end{bmatrix}$, $b = \begin{bmatrix} \tilde{b} \\ b_m \end{bmatrix}$, $(\eta, \eta_b)$ are the same as those in the original problem.

The original problem can be solved by introducing the SVD of $A$. Naturally, we can get the solutions to problem 1 and 2 by using the method of updating and downdating the singular value decomposition of $A$. Considering the particularity of updating SVD and the particularity of original problem, we develop an efficient method to solve problem 1 in section 4. A similar method for problem 2 is described in section 5. Numerical results are given in section 6.