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GLOBAL ATTRACTOR AND INERTIAL MANIFOLDS FOR REGENERATION OF SEVERED LIMB EQUATION

Xu Zhenyuan
(Wuxi Institute of Light Industry, Wuxi 214036)
Liu Zengrong
(Dept. of Math. Suzhou University, Suzhou 215006)
(LNM, Institute of Mechanics, Chinese Academy of Sciences)
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Abstract In this paper the existence of the compact global attractor and inertial manifolds for regeneration of severed limb equation are proved.

Key Words Global attractor; inertial manifold; nonlinear evolutionary equation.
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1. Introduction

The theory of inertial manifolds for nonlinear evolutionary equations has been established in [1]. It is showed that if a dissipative partial differential equation has a compact global attractor $\mathcal{A}$ with finite Hausdorff and Fractal dimensions, then it possesses a finite dimensional inertial manifold under the certain conditions, in this case partial differential equation can be reduced to ordinary differential equation on inertial manifold.

In this paper we investigate a partial differential equation which is given in the problem of regeneration of an animal in biology [2], we call it regeneration of severed limb equation, this equation is a reaction-diffusion equation of complex value function, we can write it as a reaction-diffusion equations of real functions. In Sections 2, 3 we show that the equation possesses a compact global attractor $\mathcal{A}$ with finite Hausdorff and Fractal dimensions. In Section 4 the existence of the inertial manifold is proved, when $n = 1$ more explicit dimensions estimates are given. The useful results are offered for biologist.

2. Global Attractor

We investigate the following equation:

$$\frac{\partial u}{\partial t} - \Delta u = \alpha u - \beta |u|^2 u, \quad \alpha > 0, \beta > 0 \tag{2.1}$$

where $u = u_1 + u_2 i, i = \sqrt{-1}$, (2.1) can be written as

$$\begin{cases} \dfrac{\partial u_1}{\partial t} - \Delta u_1 = \alpha u_1 - \beta(u_1^2 + u_2^2)u_1 & (2.2)_1 \\ \dfrac{\partial u_2}{\partial t} - \Delta u_2 = \alpha u_2 - \beta(u_1^2 + u_2^2)u_2 & (2.2)_2 \end{cases}$$

the initial datum is

$$u_1(x,0) = u_{10}(x), \quad u_2(x,0) = u_{20}(x) \tag{2.3$_0$}$$

Let $x \in \Omega \subset R^n$ be an open bounded region with bound Γ and boundary conditions are

$$u_i = 0 \qquad \text{on } \Gamma, \quad i = 1, 2 \tag{2.3$_1$}$$

$$\frac{\partial u_i}{\partial \nu} = 0 \qquad \text{on } \Gamma \tag{2.3$_2$}$$

$$\Omega = (0, L)^n, \ u_i \text{ are periodic functions} \tag{2.3$_3$}$$

Let $L^p(\Omega)$ be Banach space and $H^k(\Omega)$ be Sobolev space. The scalar product and norm on $L^2(\Omega)$ will be $(\cdot,\cdot)$ and $|\cdot|$, the norm on $H_0^1(\Omega)$ is written by $\|u\| = \left(\sum_{i=1}^n \left|\frac{\partial u}{\partial x_i}\right|^2\right)^{\frac{1}{2}}$. For simplicity, let

$$I\!L^p(\Omega) = L^p(\Omega) \times L^p(\Omega), \quad H = L^2(\Omega) \times L^2(\Omega)$$

$$V = \begin{cases} H_0^1(\Omega) \times H_0^1(\Omega), & \mu = 1 \\ H^1(\Omega) \times H^1(\Omega), & \mu = 2 \\ H_p^1(\Omega) \times H_p^1(\Omega), & \mu = 3 \end{cases}$$

Proposition 2.1 *For $\{u_{10}, u_{20}\} \in H$, there exists a unique solution $\{u_1, u_2\}$ of (2.2), $(2.3)_0$, $(2.3)_\mu$, it satisfies*

$$\{u_1, u_2\} \in C(I\!R^+, H)$$

$$\{u_1, u_2\} \in L^2(0, T; V) \cap L^{2p}(0, T; I\!L^{2p}(\Omega)), \quad T > 0$$

The mapping $\{u_{10}, u_{20}\} \to \{u_1(t), u_2(t)\}$ is continuous on H and defines a semigroup $\{s(t)\}$.

Proof We can apply the Galerkin procedure as [5], here it is omitted.

We will first prove the existence of the global attractor. Multiply $(2.2)_1$ by u_1, $(2.2)_2$ by u_2, integrate over Ω, we obtain

$$\begin{cases} \dfrac{1}{2}\dfrac{d|u_1|^2}{dt} - (\Delta u_1, u_1) = \alpha(u_1, u_2) - \displaystyle\int_\Omega \beta(u_1^2 + u_2^2)u_1^2 dx \\ \dfrac{1}{2}\dfrac{d|u_2|^2}{dt} - (\Delta u_2, u_2) = \alpha(u_2, u_2) - \displaystyle\int_\Omega \beta(u_1^2 + u_2^2)u_2^2 dx \end{cases} \tag{2.4}$$

by Young's inequality,

$$\int_\Omega v^2 dx \le C_0 \int_\Omega v^4 dx + C_1$$

we obtain

$$|u_1(t)|^2 + |u_2(t)|^2 \le [|u_1(0)|^2 + |u_2(0)|^2]\exp(-2\delta t) + \frac{2\beta C_1}{\delta C_0}[1 - \exp(-2\delta t)] \tag{2.5}$$