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INTERACTION OF CONORMAL WAVES WITH DIFFERENT SINGULARITIES FOR SEMI-LINEAR EQUATIONS*

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Abstract We Consider a solution of the semi-linear partial differential equations in higher space dimensions. We show that if there exist two characteristic hypersurfaces bearing different weak singularities intersect transversally, and another one characteristic hypersurface issues from above intersection, then the solution would be conormal with respect to the union of these surfaces, and satisfy the so-called "sum law".

Key Words Semi-linear equation; space of conormal distribution; different weak singularities.
Classification 35L

0. Introduction

Recently, many works have been done in the study of the conormal waves with weak singularities for semi-linear, even quasi-linear systems in higher space dimensions, namely their propagation, interaction and reflection ([1]–[7]). In [3]–[4], the interaction of conormal waves with same singularities has been considered. On the other hand, in one space dimension case, the interaction of two progressive waves with different singularities would produce "anomalous" singularities which satisfy the "sum law" as shown in [8].

In this paper, we consider the interaction of two conormal waves with different weak singularities in higher space dimensions. We will show that if two characteristic hypersurfaces bearing different weak singularities intersect transversally, and another one characteristic hypersurface issues from the intersection, then the solution would be conormal with respect to the union of these surfaces, and satisfy the "sum law".

The main result in this paper is discribed in Section 1, in Section 2 we will introduce and discuss a new space of conormal distribution. The proof of the main theorem is completed in Section 3.

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1. Notation and Statement of the Results

Let us consider a solution u of the partial differential equation of semi-linear

$$P(x,\partial_x)u(x) = F(x,u(x),\cdots,\nabla^{m-1}u(x)) \tag{1.1}$$

Where P is a differential operator of order m with smooth coefficients in R^n, and F is a smooth function with respect to $x,u,\cdots,\nabla^{m-1}u$.

Let $x=(x',t)$ and $\Omega(\subset R^n)$ be an open neighborhood which contains the origin. We denote by $p(t,x';\tau,\xi)$ the principal symbol of $P(x,\partial_x)$, and assume that

(H.1) The real roots of $p(t,x';\tau,\xi)=0$ in τ are all simple.

(H.2) If $(t_0,x_0';\tau_0,\xi_0')\in\Omega\times(R^n\setminus\{0\})$, and $p(t_0,x_0';\tau_0,\xi_0')=0$, then one of the two half-bicharacteristics issuing from $(t_0,x_0';\tau_0,\xi_0')$ must come into $\Omega_-=\Omega\cap\{t<0\}$ before it goes out the open neighborhood Ω.

(H.3) Let Σ_1,Σ_2 be characteristic hypersurfaces for P, which intersect transversally along the manifold Γ, and $\Gamma\cap\Omega_-=\emptyset$. There is exactly another one characteristic hypersurface through Γ.

Let S be a smooth surface, the space of conormal distribution $H^{s,k}(\Omega,S)$ is the set of those $u\in H^s_{\rm loc}(\Omega)$ such that $Z^Iu\in H^s_{\rm loc}(\Omega)$ for all $Z^I=Z_{i_1}\cdots Z_{i_{|I|}}$ with $|I|\le k$, $Z_j(j=1,\cdots,|I|)$ are C^∞ vector fields in R^n tangent to surface S(see [3], [4]). With these notations we can state our principal result.

Theorem 1.1 *Let $u\in H^s_{\rm loc}(\Omega)(s>\frac{n}{2}+m)$ be a solution of* (1.1). *Suppose that $u\in H^{s+l_j,k_j}(\Omega_-,\Sigma_j)(l_j+k_j=k,\ k_j\ge0,\ l_j\ge0,\ j=1,2)$ near Σ_j, and $u\in H^{s+k}_{\rm loc}(\Omega_-)$ except on $\Sigma_1\cup\Sigma_2$. Then*

i) *$u\in H^{s+l_j,k_j}(\Omega,\Sigma_j)$ near $\Sigma_j(j=1,2)$.*

ii) *$u\in H^{s+k}_{\rm loc}(\Omega)$ away from $\Sigma_1\cup\Sigma_2\cup\Sigma_3^+$, in this statement $\Sigma_3^+=\Sigma_3\setminus\Sigma_3^-$, where Σ_3^- is one connected component of $\Sigma_3\setminus\Gamma$, and $\Sigma_3^-\cap\Omega_-\ne\emptyset$.*

iii) *Let $t_0=2s+l_1+l_2-\frac{n}{2}-m+1$, if $t_0<s+k$, then $u\in H^{t_0,k_3}(\Omega)$ near $\Sigma_3^+(k_3=[s+k-t_0]$, [a] denote the integer part of a). If $t_0\ge s+k$, then $u\in H^{s+k}_{\rm loc}(\Omega)$ near Σ_3^+.*

Remark 1.2 In this paper, we consider the case where only three (real) characteristic hypersurfaces intersect on Γ. In the general case, we can discuss it by the similar methods of this paper and [4]. In fact, we can also introduce distributions conormal with three index as shown in Section 2. On the other hand, where more than three hypersurfaces $\Sigma_1,\cdots,\Sigma_N$ have to be considered, the vector fields do not suffice to describe conormal distributions to $\Sigma_1\cup\cdots\cup\Sigma_N$, and fully pseudo-differential operators have to be considered (see [4]).

We now will recall some result (see [3]) which will be used in this paper.

Lemma 1.3 *If $u\in H^s(\Omega)(s>\frac{n}{2}+m-1)$ is a solution of* (1.1) *then*

i) *For all non-characteristic points (x_0,ξ_0), u belongs to $H^{2s-\frac{n}{2}-m+2}$ microlocally.*