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ON THE GENERALIZED SYSTEM OF FERRO-MAGNETIC CHAIN WITH GILBERT DAMPING TERM

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Abstract In this paper we have established the existence of global weak solutions and blow-up properties for the generalized system of ferro-magnetic chain with Gilbert damping term by means of Galerkin method and concavity argument. In addition, the convergence as $\alpha\to 0$ and $\varepsilon\to 0$ have also been discussed.

Key Words existence; blow-up; asymptotic behavior.
Classifications 35Q10; 35B40.

0. Introduction

The evolution of spin fields in continuum ferromagnets is described by Landau-Lifshitz equations

$$M_t = -\alpha M\times(M\times H^e)+\beta M\times H^e \tag{1}$$

which bears a fundamental role in the understanding of nonequilibrium magnetism, where the magnetic field $H^e = H+\gamma\triangle M$, α,β,γ are constants with $\alpha>0$. The first term in the right hand side of (1) is called Landau-Lifshitz-Gilbert or simply Gilbert damping term.

Let $\Omega\subset R^3$ be a bounded open domain with boundary $\partial\Omega\in C^2$. The generalized system of ferromagnetic chain

$$Z_t = -\alpha Z\times(Z\times\triangle Z)+\beta Z\times\triangle Z+f(x,t,Z) \tag{2}$$

is obviously a nonlinear degenerate parabolic system of 3-dimensional vector value function $Z=(u,v,w)$, where $f(x,t,Z)$ is a given 3-dimensional vector valued function with $t\in R^+$ and $x,Z\in R^3$.

If $\alpha=0,\beta=1$, i. e. for the following system

$$Z_t = Z\times\triangle Z+f(x,t,Z) \tag{3}$$

there are some works e. g. [2—8] concerning the global existence of weak solutions for various boundary value problems and initial value problem.

In Part I of the paper we consider the homogeneous boundary value problem

$$(P_\varepsilon)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z)+\beta Z\times\triangle Z + f(x,t,Z) & (4)\\ Z|_{\partial\Omega}=0 & (5)\\ Z(x,0)=\varphi(x) & (6)\end{cases}$$

where $\varepsilon\geqslant 0$ is a constant. We have established the existence of golbal weak solutions for the problem by means of Galerkin method. In addition, the convergence, as $\varepsilon\to 0$ and $\alpha\to 0$, of the weak solutions have also been discussed in this part. Part Ⅱ is devoted to the blow-up properties for the following problem

$$(\overline{P}_\varepsilon)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z)+\beta Z\times\triangle Z + |Z|^p Z\\ Z|_{\partial\Omega}=0\\ Z(x,0)=Z_0(x)\end{cases}$$

where the constants $p>0, \varepsilon>o$.

Part I. Global Existence for the Problem (P_ε)

We shall employ Galerkin′ s method to show the existence of weak solutions for the problem (P_ε). For this purpose, we first deal with an auxiliary problem

$$(P_\varepsilon^*)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z)+\beta Z\times\triangle Z + F(x,t,Z) & (4^*)\\ (5),(6)\end{cases}$$

where the function $F(x,t,Z)$ is made as following:

$$F(x,t,Z)=\eta(Z)f(x,t,Z) \tag{7}$$

where $\eta(Z)$ is C^1 cut-off function such that $0\leqslant\eta(Z)\leqslant 1$ for any $Z\in R^3$, and

$$\eta(Z)=\begin{cases}1, & \text{if } |Z|<M_0\\ 0, & \text{if } |Z|\geqslant 2M_0\end{cases}$$

where M_0 is a positive constant to be determined in Section 3.

Let $W_n(x)$ be the eigenfunctions of the problem

$$\triangle W+\lambda_n W=0,\quad W|_{\partial\Omega}=0$$