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Bifurcation Method to Analysis of Traveling Wave Solutions for (3+1)-Dimensional Nonlinear Models Generated by the Jaulent-Miodek Hierarchy

RAN Yanping[1,2] and LI jing[1,*]

[1] *College of Applied Science, Beijing University of Technology, Beijing 100124, China.*
[2] *Tianshui Normal University, Gansu 741001, China.*

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Abstract. In this paper, the third model of four (3+1)-dimensional nonlinear evolution equations, generated by the Jaulent-Miodek hierarchy, is investigated by the bifurcation method of planar dynamical systems. The 2-parameters different bifurcation regions are obtained. According to the different phase portraits in 2-parameters different bifurcation regions, we obtain kink (anti-kink) wave solutions, solitary wave solutions and periodic wave solutions for the third of these models in the different subsets of 4-parameters space by dynamical system method. Furthermore, the explicit exact expressions of these bounded traveling waves are obtained. All these wave solutions are characterized by distinct physical structures.

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Key Words: Nonlinear (3+1)-dimensional equation; Bifurcation method; traveling wave solution.

1 Introduction

Nonlinear partial differential equations are very wildly used in many fields such as physics, engineering, mechanics, biology, chemistry economic and so on. The traveling wave solutions of nonlinear wave equations play a major role in the study of the propagation of waves and the structures of the obtained wave solutions.

*Corresponding author. *Email addresses:* `ranypmath@163.com` (Y.P. Ran), `ijing@bjut.edu.cn` (J. Li)

Four $(2+1)$-dimensional nonlinear models generated by the Jaulent-Miodek hierarchy [1–4] are extended in [5], which are

$$
\begin{aligned}
w_t &= -(w_{xx}-2w^3)_x - \frac{3}{2}(w_x\partial_x^{-1}w_y + ww_y) + \alpha\partial_x^{-1}w_{zz},\\
w_t &= \frac{1}{2}(w_{xx}-2w^3)_x - \frac{3}{2}(-\frac{1}{4}\partial_x^{-1}w_{yy} + ww_y) + \alpha\partial_x^{-1}w_{zz},\\
w_t &= \frac{1}{4}(w_{xx}-2w^3)_x - \frac{3}{4}(\frac{1}{4}\partial_x^{-1}w_{yy} + w_x\partial_x^{-1}w_y) + \alpha\partial_x^{-1}w_{zz},\\
w_t &= 2(w_{xx}-2w^3)_x - \frac{3}{4}(\partial_x^{-1}w_{yy} - 2w_x\partial_x^{-1}w_y - 6ww_y)\\
&\quad -\frac{3}{4}\partial_x^{-1}w_{zz} - \frac{1}{4}w_z - \frac{1}{2}w_y,
\end{aligned}
\tag{1.1}
$$

where α is a constant,∂_x^{-1} is the inverse of ∂_x with $\partial_x\partial_x^{-1} = \partial_x^{-1}\partial_x = I$,

$$
\partial_x^{-1} = \int_{-\infty}^{x} f(t)\mathrm{d}t. \tag{1.2}
$$

It is obvious that these $(3+1)$-dimensional nonlinear models (1) are developed by adding $\alpha\partial_x^{-1}w_{zz}$, and $-\frac{3}{4}\partial_x^{-1}w_{zz} - \frac{1}{4}w_z - \frac{1}{2}w_y$ to the $(2+1)$-dimensional nonlinear models [1]. The system(1) is completely integrable evolution equations. There are many methods to be used in travel wave soutions of nonlinear evolution equations, such as the inverse scattering method, the B*ä*cklund transformation method, algebraic-geometric method, the Darboux transformation method, multiple exp-function method [6], the Hirota bilinear method [1–3, 5, 7–9] and dynamical systems method [4, 10–12]. The Hirota bilinear method is used to formally derive the multiple kink solutions and multiple singular kink solutions of the $(2+1)$-dimensional nonlinear models [1], and multiple soliton solutions for the system (1) [5]. By the bifurcation method of the dynamical systems, some new exact solutions of the $(2+1)$-dimensional nonlinear models are obtained in [4]. In this paper, we will study the third model given

$$
w_t = -\frac{1}{4}(w_{xx}-2w^3)_x - \frac{3}{4}(\frac{1}{4}\partial_x^{-1}w_{yy} + w_x\partial_x^{-1}w_y) + \alpha\partial_x^{-1}w_{zz}, \tag{1.3}
$$

by the method of dynamical systems.

Using the potential

$$
w(x,y,t) = u_x(x,y,t) \tag{1.4}
$$

to remove the integral term in (1.3), the system (1.3) becomes

$$
u_{xt} + \frac{1}{4}u_{xxxx} - \frac{3}{2}u_x^2u_{xx} + \frac{3}{16}u_{yy} + \frac{3}{4}u_{xx}u_y - \alpha u_{zz} = 0. \tag{1.5}
$$

We are interesting in the wave solutions of (1.5) in this paper. Let $u(x,y,z,t) = \Psi(kx + ry + z - ct) = \Psi(\xi)$, where c is propagating wave velocity. According to physical meaning