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Seiberg-Witten-Like Equations Without Self-Duality on Odd Dimensional Manifolds

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Abstract. In this paper, Seiberg-Witten-like equations without self-duality are defined on any smooth $2n+1$-dimensional $Spin^c$ manifolds. Then, a non-trivial solution is given on the strictly-Pseudoconvex CR-5 manifolds endowed with a canonical $Spin^c$-structure by using Dirac operator associated with the generalized Tanaka-Webster connection. Finally, some bounds are given to them on the 5-dimensional Riemannian manifolds.

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1 Introduction

Recently, Seiberg-Witten theory has played an important role in the topology of 4-manifolds. One of the important part of this theory is Seiberg-Witten equations consist of curvature equation and Dirac equation. Dirac equation can be written down on any $Spin^c$-manifold in any dimension. Due to the self-duality of a 2-form, the curvature equation is special to 4-dimensional manifolds. There are some generalizations of these equations to higher dimensional manifolds. All of them are mainly based on the generalized self-duality of a 2-form [1,2].

A global solution to these equations is given by means of the canonical $Spin^c$-structure. Since any almost Hermitian manifold has a canonical $Spin^c$-structure which is determined by its almost Hermitian structure, a fundemantal role is played by the almost Hermitian manifold. As in almost Hermitian manifolds, every contact metric manifold

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can be equipped with the canonical $Spin^c$-structure determined by the almost complex structure on the contact distribution. Also, on these manifolds, one can described a spinor bundle. Therefore, for a given canonical $Spin^c$-structure on the contact metric manifold, a spinorial connection can be defined on the associated spinor bundle by means of the generalized Tanaka-Webster connection [3]. Then, on any contact metric manifolds endowed with $Spin^c$-structure, Dirac operator is associated to a such connection. Also, Kohn-Dirac operator is defined by restriction of the contact metric manifold to the contact distribution. By using this Dirac operator we define Dirac equation on a $2n+1$-dimensional contact metric manifold. Also, curvature equation is written down without using self-duality concept on any (2n+1)-dimensional contact metric manifold via orthogonal projection of the endomorphisms onto a particular subbundle.

The plan of this paper is in the following. In Section 2, some basic facts concerning the contact metric manifolds and CR manifolds are written. Then canonical $Spin^c$-structure and Dirac operator are given by means of the generalized Tanaka-Webster connection. In Section 3, Seiberg-Witten-like equations without self-duality are defined on (2n+1)-dimensional contact metric manifold by means of the generalized Tanaka-Webster conection. Then, in particular, a non-trivial solution is given on the 5-dimensional strictly-Pseudoconvex CR manifolds. In the final section, some bounds are given to the solution of these equations on the 5-dimensional Riemannian manifolds.

2 Some basic materials

2.1 $Spin^c$ manifolds

Assume that M is an $2n+1$ dimensional orientable Riemannian manifold. Then the definitions of the $Spin^c$structures on M are obtained as follows:

Accordingly, the structure group of M is $SO(2n+1)$ and there is an open covering $\{U_\alpha\}_{\alpha\in A}$ with the transition functions $g_{\alpha\beta}: U_\alpha\cap U_\beta \longrightarrow SO(2n+1)$ for M. In addition, if there exists another collection of transition functions

$$\widetilde{g}_{\alpha\beta}: U_\alpha\cap U_\beta \longrightarrow Spin^c(2n+1)$$

such that the following diagram commutes

$$\begin{tikzcd} & Spin^c(2n+1) \arrow[d, "\lambda"] \\ U_\alpha\cup U_\beta \arrow[ur, "\widetilde{g}_{\alpha\beta}"] \arrow[r, "g_{\alpha\beta}"'] & SO(2n+1) \end{tikzcd}$$

that is, $\lambda\circ\widetilde{g}_{\alpha\beta}=g_{\alpha\beta}$ and the cocycle condition $\widetilde{g}_{\alpha\beta}(x)\circ\widetilde{g}_{\beta\gamma}(x)=\widetilde{g}_{\alpha\gamma}(x)$ on $U_\alpha\cap U_\beta\cap U_\gamma$ is satisfied, then M is called $Spin^c$ manifold. Then on a $Spin^c$ manifold M, one can construct three principal bundles [4].