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On Solving the (2+1)-Dimensional Nonlinear Cubic-Quintic Ginzburg-Landau Equation Using Five Different Techniques

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Abstract. In this article, we apply five different techniques, namely the (G'/G)-expansion method, an auxiliary equation method, the modified simple equation method, the first integral method and the Riccati equation method for constructing many new exact solutions with parameters as well as the bright-dark, singular and other soliton solutions of the (2+1)-dimensional nonlinear cubic-quintic Ginzburg-Landau equation. Comparing the solutions of this nonlinear equation together with each other are presented. Comparing our new results obtained in this article with the well-known results are given too.

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Key Words: (G'/G)-expansion method; auxiliary equation method; modified simple equation method; first integral method; Riccati equation method; exact traveling wave solutions; solitary wave solutions; Cubic-quintic Ginzburg-Landau equation.

1 Introduction

Investigations of exact traveling wave solutions, solitary wave solutions, singular solitary wave solutions and periodic wave solutions to nonlinear evolution equations play an

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important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical physics and so on. In recent decades, many effective methods have been established to obtain the exact traveling wave solutions of the nonlinear evolution equations in mathematical physics, such as the inverse scattering transform [1], the Hirota method [2], the truncated Painlevé expansion method [3], the Bäcklund transform method [1,4,5], the exp-function method [6-8], the simplest equation method [9,10], the Weierstrass elliptic function method [11], the Jacobi elliptic function method [12-16], the tan-function method [17-22], the sine-cosine method [23,24], the (G'/G)-expansion method [25-30], the modified simple equation method [31-36], the Kudryashov method [37-39], the multiple exp-function method [40,41], the transformed rational function method [42], a generalized new auxiliary equation method [43], an extended auxiliary equation method [44,45], the $(G'/G,1/G)$-expansion method [46-50], the first integral method [51,52], the soliton ansatz method [54-69], a special kind of (G'/G)-expansion method [70-74], a new mapping method [75], a new extended auxiliary equation method [76], the extended trial equation method [77-81], and so on.

The objective of this article is to apply five effective methods, namely, the (G'/G)-expansion method, an auxiliary equation method, the modified simple equation method, the first integral method and the Riccati equation method for solving the following (2+1)-dimensional nonlinear cubic-quintic Ginzburg-Landau equation [8,45,50,76]:

$$iu_z+\frac{1}{2}u_{xx}+\frac{1}{2}(\beta-i)u_{\tau\tau}+iu+(1-ir_1)u|u|^2+ir_2u|u|^4=0,\quad i=\sqrt{-1}, \tag{1.1}$$

where the complex function $u(x,z,\tau)$ is slowly varying envelope of the electric field, β is a real constant, z and x are the propagation and transverse coordinates respectively, while r_1 and r_2 are constants. More over,

$$\tau=t-\frac{z}{V_0}$$

is the so-called reduced time, where t is the physical time, and V_0 is the group velocity of the carrier wave. Eq. (1.1) has been discussed in [8] using the exp-function method, in [50] using $(G'/G,1/G)$-expansion method, in [45] using an extended auxiliary equation method and in [76] using a new extended auxiliary equation method where its exact solutions have been obtained. To our knowledgement, Eq. (1.1) is not investigated elsewhere using the five methods proposed in this article.

This article is organized as follows: In Sections 2,3,4, we give the description of the (G'/G)-expansion method, the modified simple equation method and the first integral method, respectively. In Section 5, we solve Eq. (1.1) using the five suggested methods. In Section 6, some graphical representations of our results are presented. In Section 7, some conclusions are obtained.