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Neutral Fractional Stochastic Differential Equations Driven by Rosenblatt Process

XU Liping and LI Zhi*

School of Information and Mathematics, Yangtze University, Jingzhou 434023, China.

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Abstract. In this paper, we are concerned with a class of neutral fractional stochastic partial differential equations driven by a Rosenblatt process. By the stochastic analysis technique, the properties of operator semigroup and combining the Banach fixed-point theorem, we prove the existence and uniqueness of the mild solutions to this kind of equations driven by Rosenblatt process. In the end, an example is given to demonstrate the theory of our work.

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1 Introduction

The long-memory and long range-dependence property of fractional Brownian motion (fBm) B^H, with Hurst parameter $H\in(1/2,1)$ make the fBm as a potential candidate to model noise in biology [1]; in mathematical finance [2]; in the analysis of global temperature anomaly [3]; in electricity markets [4] and communication networks [5] etc.

However, the fBm belongs to the family of Hermite processes which admit the following representation, for all $d\geq 1$:

$$Y_t^{H,d}=c(H_0)\int_{\mathbb{R}}\cdots\int_{\mathbb{R}}\Big(\int_0^t \Pi_{j=1}^d (s-x_j)_+^{H_0-1}\mathrm{d}s\Big)\mathrm{d}B_{x_1}\cdots \mathrm{d}B_{x_d},\qquad \forall t>0,$$

where $\{B_x: x\in\mathbb{R}\}$ is a two-sided Brownian motion, $c(H_0)$ is a normalizing constant such that $\mathbb{E}|Y_1^{H,d}|^2=1$ and $H_0=\frac{1}{2}+\frac{H-1}{d}$ with $H\in(\frac{1}{2},1)$. When $d=1$, Hermite process is the fractional Brownian motion with Hurst parameter $H\in(1/2,1)$.

*Corresponding author. *Email addresses:* `lizhi_csu@126.com` (Z. Li), `xlp211@126.com` (L. P. Xu)

When $d=2$, the Hermite process is called the Rosenblatt process by Taqqu [6]. The Rosenblatt process share many properties with fBm. For example, they are self-similar processes with Hurst parameter H and have stationary increments [7]. They exhibit long range-dependence and their sample paths are almost-surely Hölder continuous of order strictly less than H. In addition, Albin [8], Veillette and Taqqu [9] and Maejima and Tudor [10] have studied the distributional properties of Rosenblatt process. Geometric properties the Rosenblatt process have been considered in [11]. But, as far as we know, in contrast to the extensive studies on fractional Brownian motion, there has been little systematic investigation on other Hermite processes. The main reasons for this, in our opinion, are the complexity of the dependence structures and the property of non-Gaussianity. Therefore, it seems interesting to study the Rosenblatt process.

On the other hand, fractional calculus and fractional differential equations have attracted the attention of many researchers due to their important applications to problems in mathematical physics, chemistry, biology and engineering. Many results on existence and stability of solutions to various type of fractional differential equations has been obtained. For more details on this topic, one can refer to [12–14]. The deterministic models often fluctuate due to noise. Systems are often subjected to random perturbations. With the help of semigroup theory and fractional calculus technique, some authors have also considered fractional (neutral) stochastic differential equations. Ones can refer to the literatures [15–18].

To the best of our knowledge, there are no paper which studied the fractional stochastic differential equations driven by Rosenblatt process. To close the gap, we will make the first attempt to study the following fractional stochastic differential equations driven by Rosenblatt processes in this paper.

$$\begin{cases} {}^CD_t^q[x(t)-g(t,x(t-r(t)))]=Ax(t)+f(t,x(t-\rho(t)))+\sigma(t)\dfrac{\mathrm{d}Z_Q^H(t)}{\mathrm{d}t}, & t\in[0,T], \\ x(t)=\xi(t)\in C([-\tau,0],U), \qquad t\in[-\tau,0], \end{cases} \tag{1.1}$$

where the fractional derivative ${}^CD^q$, $q\in(1/2,1)$, is understood in the Caputo sense, $Z_Q^H(t)$ is a Rosenblatt process with parameter $H\in(\frac{1}{2},1)$ in a real and separable Hilbert space $(K,\|\cdot\|_K,\langle\cdot,\cdot\rangle_K)$, $\tau>0$ and A generates a strongly continuous semigroup $\{S(t)\}_{t\geq 0}$ in a Hilpert space U with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. Here $C([-\tau,0];U)$ denote the family of all continuous U-valued functions ξ from $[-\tau,0]$ to U with the norm $\|\xi\|_C=\sup_{-\tau\leq t\leq 0}\|\xi(t)\|$, $r,\rho:[0,+\infty)\to[0,\tau]$ are continuous functions, $g,f:[0,+\infty)\times U\to U$ are two given measurable mappings and σ is a given function to be specified later. The main aim of this paper is to investigate the existence and uniqueness of the mild solution to (1.1) by using the stochastic analysis techniques, the properties of operator semigroup and combining the fixed point theorem.

The rest of this paper is organized as follows. In Section 2, we introduce some necessary notations and preliminaries. In Section 3, we devote to investigate the existence and