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Control and Stabilization of High-Order KdV Equation Posed on the Periodic Domain

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Abstract. In this paper, we study exact controllability and feedback stabilization for the distributed parameter control system described by high-order KdV equation posed on a periodic domain $\mathbb{T}$ with an internal control acting on an arbitrary small nonempty subdomain ω of $\mathbb{T}$. On one hand, we show that the distributed parameter control system is locally exactly controllable with the help of Bourgain smoothing effect; on the other hand, we prove that the feedback system is locally exponentially stable with an arbitrarily large decay rate when Slemrod's feedback input is chosen.

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1 Introduction

In this paper, we will investigate the following higher-order dispersive equation posed on the periodic domain $\mathbb{T}$ (a unit circle in the plane) from the control point of view:

$$\partial_t u+(-1)^{l+1}\partial_x^{2l+1}u+u\partial_x u=f, \quad x\in\mathbb{T},\ t\in\mathbb{R}, \tag{1.1}$$

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where f is the control input supported in a given open set $\omega \subset \mathbb{T}$. The assumption on the periodic domain is equivalent to impose the periodic boundary conditions over the interval $(0,2\pi)$:

$$\partial_x^n u(0,t) = \partial_x^n u(2\pi,t), \quad n = 0,1,\cdots,2l.$$

The following two fundamental control theory problems will be discussed:

Exact controllability: *For the given initial state u_0 and terminal state u_1 belong in a certain space, can one find an appropriate control input f such that equation* (1.1) *admits a solution u which satisfies*

$$u|_{t=0} = u_0, \quad u|_{t=T} = u_1?$$

Feedback stabilization: *Is there a feedback control law: $f = Ku$ such that the resulting closed-loop system*

$$\partial_t u + (-1)^{l+1}\partial_x^{2l+1} u + u\partial_x u = Ku, \quad x \in \mathbb{T},\ t \in \mathbb{R}$$

is exponentially stable as $t \to \infty$?

Since for the solution of (1.1) satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{T}} u(x,t)\mathrm{d}x = \int_{\mathbb{T}} f(x,t)\mathrm{d}x,$$

the mass will be conserved provided that

$$\int_{\mathbb{T}} f(x,t)\mathrm{d}x = 0.$$

For the purpose of mass conservation, the control input as follows is chosen (see [5]):

$$f(x,t) = [Gh](x,t) := g(x)\left(h(x,t) - \int_{\mathbb{T}} g(y)h(y,t)\mathrm{d}y\right) \tag{1.2}$$

where $g(x)$ is a given nonnegative smooth function such that $\{g > 0\} = \omega \subset \mathbb{T}$ and

$$2\pi[g] = \int_{\mathbb{T}} g(x)\mathrm{d}x = 1.$$

With h as a new control input, the resulting control system turns to be

$$\partial_t u + (-1)^{l+1}\partial_x^{2l+1} u + u\partial_x u = Gh, \quad x \in \mathbb{T},\ t \in \mathbb{R}. \tag{1.3}$$

We state the main results as follows:

Theorem 1.1 (Exact controllability)**.** *Let $T > 0$ and $s \geq s_0$ (see Lemma* 3.2*) be given. Then there exists a $\delta > 0$ such that for any u_0, $u_1 \in H^s(\mathbb{T})$ with $[u_0] = [u_1]$ and*

$$\|u_0\|_{H^s(\mathbb{T})} \leq \delta, \qquad \|u_1\|_{H^s(\mathbb{T})} \leq \delta.$$