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Solitons and Other Solutions for the Generalized KdV–mKdV Equation with Higher-order Nonlinear Terms

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Abstract. The generalized sub-ODE method, the rational (G'/G)-expansion method, the exp-function method and the sine-cosine method are applied for constructing many traveling wave solutions of nonlinear partial differential equations (PDEs). Some illustrative equations are investigated by these methods and many hyperbolic, trigonometric and rational function solutions are found. We apply these methods to obtain the exact solutions for the generalized KdV-mKdV (GKdV-mKdV) equation with higher-order nonlinear terms. The obtained results confirm that the proposed methods are efficient techniques for analytic treatment of a wide variety of nonlinear partial differential equations in mathematical physics. We compare between the results yielding from these methods. Also, a comparison between our new results in this paper and the well-known results are given.

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Key Words: Generalized sub-ODE method; rational (G'/G)-expansion method; exp-function method; sine-cosine method; generalized KdV-mKdV equation with higher-order nonlinear terms; exact solutions; solitary wave solutions.

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1 Introduction

Nonlinear partial differential equations (PDEs) are widely used to describe many important phenomena and dynamic processes in physics, chemistry, biology, fluid dynamics,

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plasma, optical fibers and other areas of engineering. Many efforts have been made to study these nonlinear equations. One of the most exciting advances of nonlinear science and theoretical physics has been developed methods that look for exact solutions for these nonlinear equations. The availability of symbolic computations such as Mathematica or Maple has popularized direct seeking for exact solutions of the nonlinear equations. Therefore, exact solution methods of PDEs have become more and more important resulting in methods like the exp-function method [1-7], the sine-cosine method [8-10], the homogeneous balance method [11,12], the tanh-sech method [13-17], the extended tanh-coth method [18-22], the (G'/G)-expansion method [23-27], the modified simple equation method [28-32], the multiple exp-function method [33,34], the first integral method [35,36], the generalized Kudryashov method [37,38], the symmetry method [39,40], the soliton ansatz method [41-62], the collocation spectral method [63], the sub-ODE method [64-66] and so on.

The objective of this paper is to apply the generalized sub-ODE method, the rational (G'/G)-expansion method, the exp-function method and the sine-cosine method for finding exact solutions of the GKdV–mKdV equation with higher-order nonlinear terms [64-67]:

$$u_t+\left(\alpha+\beta u^p+\gamma u^{2p}\right)u_x+u_{xxx}=0, \tag{1.1}$$

where α,β and γ are constants, while p is a positive integer. Eq. (1.1) includes three important nonlinear equations, namely, the KdV–mKdV equation, the KdV equation and the mKdV equation. Eq. (1.1) has been discussed in [64-66] using a sub-ODE method and in [67] using the (G'/G)-expansion method and its exact solutions have been found.

This paper is organized as follows: In Sections 2-5, we describe the generalized sub-ODE method, the rational (G'/G)-expansion method, the exp-function method and the sine-cosine method. In Section 6, we apply these methods to find the hyperbolic, trigonometric function solutions and rational function solutions of Eq. (1.1). In Section 7, some graphical representations for some solutions of Eq. (1.1) are presented. In Section 8, some conclusions are illustrated.

2 Description of the generalized sub-ODE method

Suppose that a nonlinear PDE has the following from:

$$P(u,u_t,u_x,u_{tt},u_{xt},u_{xx},...)=0, \tag{2.1}$$

where $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

The main steps of the generalized sub-ODE method used in our paper are different from that obtained in [64-66] which described as follows:

Step 1. First of all, we use the wave transformation:

$$u(x,t)=U(\zeta),\quad \zeta=kx\pm\lambda t, \tag{2.2}$$