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On a Linear Partial Differential Equation of the Higher Order in Two Variables with Initial Condition Whose Coefficients are Real-valued Simple Step Functions

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Abstract. By using the method developed in the paper [*Georg. Inter. J. Sci. Tech.*, Volume 3, Issue 1 (2011), 107-129], it is obtained a representation in an explicit form of the weak solution of a linear partial differential equation of the higher order in two variables with initial condition whose coefficients are real-valued simple step functions.

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1 Introduction

In [1] has been obtained a representation in an explicit form of the solution of the linear partial differential equation of the higher order in two variables with initial condition whose coefficients were real-valued coefficients. The aim of the present manuscript is resolve an analogous problem for a linear partial differential equation of the higher order in two variables with initial condition whose coefficients are real-valued simple step functions.

The paper is organized as follows.

In Section 2, we consider some auxiliary notions and facts which come from works [1–3]. In Section 3, we get a representation in an explicit form of the weak solution of

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the partial differential equation of the higher order in two variables with initial condition whose coefficients are real-valued simple step functions.

2 Some auxiliary notions and results

Definition 2.1. *Fourier differential operator $(\mathcal{F})\frac{\partial}{\partial x}$ in R^{∞} is defined as follows :*

$$(\mathcal{F})\frac{\partial}{\partial x}\begin{pmatrix}\frac{a_0}{2}\\ a_1\\ b_1\\ a_2\\ b_2\\ a_3\\ b_3\\ \vdots\end{pmatrix}=\begin{pmatrix}0&0&0&0&0&0&0&\cdots\\ 0&0&\frac{1\pi}{l}&0&0&0&0&\cdots\\ 0&-\frac{1\pi}{l}&0&0&0&0&0&\cdots\\ 0&0&0&0&\frac{2\pi}{l}&0&0&\cdots\\ 0&0&0&-\frac{2\pi}{l}&0&0&0&\cdots\\ 0&0&0&0&0&0&\frac{3\pi}{l}&\ddots\\ 0&0&0&0&0&-\frac{3\pi}{l}&0&\ddots\\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\ddots\end{pmatrix}\times\begin{pmatrix}\frac{a_0}{2}\\ a_1\\ b_1\\ a_2\\ b_2\\ a_3\\ b_2\\ \vdots\end{pmatrix}. \tag{2.1}$$

For $n\in\mathbb{N}$, let $FD^n[-l,l[$ be a vector space of all n-times differentiable functions on $[-l,l[$ such that for arbitrary $0\le k\le n-1$, a series obtained by a differentiation term by term of the Fourier series of $f^{(k)}$ pointwise converges to $f^{(k+1)}$ for all $x\in[-l,l[$.

Lemma 2.1. *Let $f\in FD^{(1)}[-l,l[$. Let G_M be an embedding of the $FD^{(1)}[-l,l[$ in to R^{∞} which sends a function to a sequence of real numbers consisting from its Fourier coefficients. i.e., if*

$$f(x)=\frac{c_0}{2}+\sum_{k=1}^{\infty}c_k\cos\left(\frac{k\pi x}{l}\right)+d_k\sin\left(\frac{k\pi x}{l}\right)\ (x\in[-l,l[),$$

then $G_F(f)=(\frac{c_0}{2},c_1,d_1,c_2,d_2,\dots)$. Then, for $f\in FD^{(1)}[-l,l[$, the following equality

$$\left(G_F^{-1}\circ(\mathcal{F})\frac{\partial}{\partial x}\circ G_F\right)(f)=\frac{\partial}{\partial x}(f) \tag{2.2}$$

holds.

Proof. Assume that for $f\in FD^{(1)}[-l,l[$, we have the following representation

$$f(x)=\frac{c_0}{2}+\sum_{k=1}^{\infty}c_k\cos\left(\frac{k\pi x}{l}\right)+d_k\sin\left(\frac{k\pi x}{l}\right)\ (x\in[-l,l[).$$

By the definition of the class $FD^{(1)}[-l,l[$, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}(f)=\frac{\partial}{\partial x}\left(\frac{c_0}{2}+\sum_{k=1}^{\infty}c_k\cos\left(\frac{k\pi x}{l}\right)+d_k\sin\left(\frac{k\pi x}{l}\right)\right)$$