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Semi-discretization Difference Approximation for a Cauchy Problem of Heat Equation in Two-dimensional Space

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Abstract. In this paper we consider a semi-descretization difference scheme for solving a Cauchy problem of heat equation in two-dimensional setting. Some error estimates are proved for the semi-descretization difference regularization method which cannot be fitted into the framework of regularization theory presented by Engl, Hanke and Neubauer. Numerical results show that the proposed method works well.

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1 Introduction

In practical many dynamic heat transfer situations, it is sometimes necessary to estimate the surface temperature or heat flux on a body from a measured temperature history at a fixed location inside the body. This so-called inverse heat conduction problem (IHCP) [1] has been investigated by many authors. It is well known that IHCP is an exponentially ill-posed problem [2]. All kinds of the regularization strategies were proposed to obtain a stable numerical solution for the problem. These include the Tikhonov regularization method [3], difference approximation method [4], wavelet method [5], Fourier cut-off method [6], hyperbolic approximation method [7-8], optimal filtering method [9], mollification methods [10-12], and optimal stable approximation methods [13]. The reader can refer to http://www.mai.liu.se/ frber/ip/index.html for more details. However, as said

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above, for the IHCP most of these authors restricted themselves to the case of the one-dimensional space. Recently IHCP in two-dimensional space has been studied by Qian and his co-works, please refer to [14-16]. In this paper we present many different difference schemes and in detail discuss a central difference scheme as a model to stabilize the ill-posed IHCP by giving the error estimates. This class of methods is important because they can easily be adapted to treat nonlinear problems that often occur in applications.

The emphasis of this work lies in the numerical approximation of the problem, the proof of the stability of the proposed difference schemes and the presentation of numerical results. We stress that the current work can be considered as an extension of [15], in which Qian and Zhang analyzed the temperature. However in this work we discussed the heat flux and implemented the numerical experiment.

2 The problem and methods

2.1 The model problem

In this paper, we consider the following Cauchy problem for heat equation in two-dimensional space [14]:

$$\begin{aligned} &u_t(x,y,t)=u_{xx}(x,y,t)+u_{yy}(x,y,t), \qquad 0<x<L,\ y\in\mathbb{R},\ t\geq 0, \\ &u(0,y,t)=g(y,t), \qquad y\in\mathbb{R},\ t\geq 0, \\ &u_x(0,y,t)=h(y,t), \qquad y\in\mathbb{R},\ t\geq 0, \\ &u(x,y,0)=u_0(x,y), \qquad 0<x<L,\ y\in\mathbb{R}. \end{aligned} \tag{2.1}$$

For the purpose of simplification, we only consider the case of $h(y,t)=0$, $u_0(x,y)=0$.

In practice, the measured data function $g^\delta(y,t)$ is available at hand.

2.2 Ill-posedness in L^2 space

In this subsection, we analyze the ill-posedness of problem (2.1) in the frequency domain. In order to use the Fourier transform technique, we extend the functions $u(x,y,t)$, $g(y,t)$, $g^\delta(y,t)$ to the whole t axis by defining them to be the zero everywhere in $t<0$. Thus we wish to determine the temperature $u(x,y,t)\in L^2(\mathbb{R}^2)$ and heat flux $u_x(x,y,t)\in L^2(\mathbb{R}^2)$ for $0<x\leq L$ from the temperature measurements $g^\delta(y,t)\in L^2(\mathbb{R}^2)$.

We also assume that these functions are in $L^2(\mathbb{R}^2)$ and use the corresponding L^2-norm. as defined below,

$$\|g\|=\left(\int_{R^2}|g(y,t)|^2\mathrm{d}y\mathrm{d}t\right)^{\frac{1}{2}}.$$