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A New Jacobi Elliptic Function Expansion Method for Solving a Nonlinear PDE Describing Pulse Narrowing Nonlinear Transmission Lines

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Abstract. In this article, we apply the first elliptic function equation to find a new kind of solutions of nonlinear partial differential equations (PDEs) based on the homogeneous balance method, the Jacobi elliptic expansion method and the auxiliary equation method. New exact solutions to the Jacobi elliptic functions of a nonlinear PDE describing pulse narrowing nonlinear transmission lines are given with the aid of computer program, e.g. Maple or Mathematica. Based on Kirchhoff's current law and Kirchhoff's voltage law, the given nonlinear PDE has been derived and can be reduced to a nonlinear ordinary differential equation (ODE) using a simple transformation. The given method in this article is straightforward and concise, and can be applied to other nonlinear PDEs in mathematical physics. Further results may be obtained.

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1 Introduction

The nonlinear PDEs in mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics, such as fluid mechanics, hydrodynamics, optics, plasma physics and so on. Recently, many methods for finding these solutions have been presented, for example, tanh-sech

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method [2-4], extended tanh-method [5-7], sine-cosine method [8-10], homogeneous balance method [11,12], Jacobi elliptic function method [13-16], F-expansion method [17-19], exp-function method [20,21], trigonometric function series method [22], $(\frac{G'}{G})$-expansion method [23-27], the modified simple equation method [28-33], the modified mapping method [34], the first integral method [35-38], the multiple exp-function algorithm method [39,40], the transformed rational function method [41], the Frobenius decomposition technique [42], the local fractional variation iteration method [43], the local fractional series expansion method [44] and so on.

The objective of this article is to use a new Jacobi elliptic function expansion method [45] to find the exact solutions of the following nonlinear PDE describing pulse narrowing nonlinear transmission lines [46]:

$$\frac{\partial^2 V(x,t)}{\partial t^2}-\frac{1}{LC_0}\frac{\partial^2 V(x,t)}{\partial x^2}-\frac{b_1}{2}\frac{\partial^2 V^2(x,t)}{\partial t^2}-\frac{\delta^2}{12LC_0}\frac{\partial^4 V(x,t)}{\partial x^4}=0, \tag{1.1}$$

where $V(x,t)$ is the voltage of the pulse and C_0, L, δ and b_1 are constants. The physical details of the derivation of Eq. (1.1) is elaborated in [46] using the Kirchhoff's current law and Kirchhoff's voltage law, which are omitted here for simplicity. It is well-known [46] that Eq. (1.1) has the solution:

$$V(x,t)=\frac{3(v^2-v_0^2)}{b_1 v^2}\operatorname{sech}^2\left[\frac{\sqrt{3(v^2-v_0^2)}}{v_0}\left(\frac{(x-vt)}{\delta}\right)\right], \tag{1.2}$$

where v is the propagation velocity of the pulse and $v_0=1/\sqrt{LC_0}$, provided that $v>v_0$.

This paper is organized as follows: In Sec. 2, the description of a new Jacobi elliptic function expansion method is given. In Sec. 3, we use the given method described in Sec. 2, to find exact solutions of Eq. (1.1). In Sec. 4, the physical explanations of some results are presented. In Sec. 5, some conclusions are obtained.

2 Description of a new Jacobi elliptic function expansion method

Consider a nonlinear PDE in the form

$$P(V,V_x,V_t,V_{xx},V_{tt},\ldots)=0, \tag{2.1}$$

where $V=V(x,t)$ is an unknown function, P is a polynomial in $V(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Let us now give the main steps of the Jacobi elliptic function expansion method [45]:

Step 1. We look for the voltage $V(x,t)$ of the pulse in the traveling form:

$$V(x,t)=V(\xi), \qquad \xi=x-vt, \tag{2.2}$$