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Generalized Frankl-Rassias Problem for a Class of Mixed Type Equations in an Infinite Domain

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Abstract. In this paper, we study the boundary-value problem for mixed type equation with singular coefficient. We prove the unique solvability of the mentioned problem with the help of the extremum principle. The proof of the existence is based on the theory of singular integral equations, Wiener-Hopf equations and Fredholm integral equations.

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1 Introduction

Boundary-value problems for mixed type equations in various unbounded domains were studied in detail in works [1-2]. The existence and the uniqueness of the solution of Frankl's problem for several mixed type equations were proved by A.V. Bitsadze [3], U.V. Devingtal [4], N.M. Fleisher [5] and by other scientists. A series of interesting results, devoted to studying boundary-value problems for partial differential equations were obtained in works [6-8]. In the work [9] a non-local problem with Bitsadze-Samarskii condition on parallel characteristics, one of which lies inside of characteristic triangle, for mixed type equation was investigated. In [10] the general Tricomi-Rassias problem was investigated for the generalized Chaplygin equation. The Tricomi problem with two parabolic lines of degeneracy, was considered in [11].

The uniqueness of quasi-regular solutions for the exterior Tricomi and Frankl problems for quaterelliptic-quaterhyperbolic mixed type partial differential equations of second order with eight parabolic degenerate lines proved by J.M. Rassias [12]. In the work

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[13] some boundary-value problems with nonlocal initial condition for model and degenerate parabolic equations with parameter were considered. The well-posedness of the extended Tricomi-Chaplygin-Frankl problem in a multidimensional region was established in [14]. The Tricomi and Frankl problems for partial differential equations were investigated by G.C. Wen [15-17]. Boundary value problems for the wave equation and equations of mixed type considered in [18].

The present work is devoted to the investigation of the problem with Frankl type condition on a segment of line of degeneration for mixed type equation with singular coefficient.

Formulation of the problem

Consider the equation

$$\operatorname{sign} y|y|^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0 \tag{1.1}$$

in the domain $D = D^+ \cup D^- \cup I$, where D^+ is a first open quadrant of the plain, D^- is a bounded domain in the fourth quadrant of the plane bounded by characteristics OC and BC of Eq. (1.1) issuing from points $O(0,0)$ and $B(1,0)$, and by segment OB of the straight line $y=0$, $I = \{(x,y): 0 < x < 1, y = 0\}$.

In Eq. (1.1) m, β_0 are some real numbers satisfying conditions $m > 0$, $-\frac{m}{2} < \beta_0 < 1$.

Introduce the following denotations:

$$I_0 = \{(x,y): 0 \leqslant y < \infty, x = 0\}, \quad I_1 = \{(x,y): 1 \leqslant x < \infty, y = 0\},$$

C_0 and C_1 are, correspondingly, points of intersection of characteristics OC and BC with characteristic issuing from the point $\mathrm{E}(c,0)$, where $c \in I$ is an arbitrary fixed value.

Let $p(x) \in C^2[0,c]$ be a diffeomorphism from the set of points of the segment $[0,c]$ to the set of points of the segment $[c,1]$ such that $p'(x) < 0, p(0) = 1$, and $p(c) = c$. As an example of such a function consider the linear function $p(x) = 1 - kx$, where $k = (1-c)/c$.

Problem TF**.** Find a function $u(x,y)$ with the following properties:

1) $u(x,y) \in C(\bar{D})$, where $\bar{D} = D \cup I_0 \cup OC \cup BC \cup I_1$;

2) $u(x,y) \in C^2(D^+)$ and satisfies Eq. (1.1) in this domain;

3) $u(x,y)$ is a generalized solution from the class R_1 (see [19]) in the domain D^-;

4)
$$\lim_{R\to\infty} u(x,y) = 0, \quad R^2 = x^2 + 4y^{m+2}/(m+2)^2, \quad x \geqslant 0, \quad y \geqslant 0; \tag{1.2}$$

5) $u(x,y)$ satisfies the boundary conditions

$$u(0,y) = \varphi(y), \quad y \in I_0, \tag{1.3}$$

$$u(x,0) = \tau_1(x), \quad x \in I_1, \tag{1.4}$$

$$u(x,y)|_{OC_0} = \psi(x), \quad 0 \leqslant x \leqslant c/2, \tag{1.5}$$

$$u(p(x),0) = \mu u(x,0) + f(x), \quad 0 \leqslant x \leqslant c, \tag{1.6}$$