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On the Cauchy Problem of the Ginzburg-Landau Equations for Atomic Fermi Gases Near the BCS-BEC Crossover

CHEN Shuhong* and GUO Boling

Institute of Applied Physics and Computational Mathematics, PO Box 8009, Beijing 100088, China.

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Abstract. In this paper we investigate time-dependent Ginzburg-Landau equations come from the superfluid atomic Fermi-gases near the Feshbach resonance from the Fermion-Boson model. We obtain the global existence and uniqueness of solution to the equations in various spatial dimensions.

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1 Introduction

In a recent study [1] it was proposed that the BCS-BEC crossover phenomena of the superfluid atomic gases could be investigated in the coupled time-dependent Ginzburg-Landau (TDGL) equations:

$$\begin{aligned}-idu_t=&\left(-\frac{dg^2+1}{U}+a\right)u+g[a+d(2\nu-2\mu)]\varphi_B+\frac{c}{4m}\triangle u+\frac{g}{4m}(c-d)\triangle\varphi_B\\&-b|u+g\varphi_B|^2(u+g\varphi_B),\end{aligned}\tag{1.1}$$

$$i\frac{\partial\varphi_B}{\partial t}=-\frac{g}{U}u+(2\nu-2\mu)\varphi_B-\frac{1}{4m}\triangle\varphi_B.\tag{1.2}$$

*Corresponding author. *Email addresses:* `shiny0320@163.com` (S. Chen), `gbl@iapcm.ac.cn` (B. Guo)

In this paper, we study the global existence and uniqueness of solutions of the coupled time-dependent Ginzburg-Landau equations for atomic Fermi gases near the BCS-BEC crossover (1.1) and (1.2) with periodic initial conditions:

$$u(x,0)=u_0(x), \quad \varphi_B(x,0)=\varphi_{B0}(x), \quad x\in\Omega, \tag{1.3}$$

$$u(x,t)=u(x+Le_i,t), \quad \varphi_B(x,t)=\varphi_B(x+Le_i,t), \quad i=1,2,\cdots, \quad x\in\Omega, \quad t\geq 0. \tag{1.4}$$

The spatial domain Ω is a bounded domain in n-dimensional Eulerian space R^n, and the time $t\geq 0$. The Fermion-pair field $u(x,t)$ and the condensed Boson field $\varphi_B(x,t)$ are L-periodic, and coupling constants a and b are real numbers, while the parameters c,m,U in (1.1) and (1.2) are assumed to be positive. Otherwise, μ is a real coefficient standing for the chemical potential, 2ν standing for the threshold energy of the Feshbach resonance is a real constant, g being the coupling constant in the Feshbach resonance is a real coefficient, d is generally complex. In the BCS-BEC crossover region both the real and imaginary parts of d have finite value. However, in the BCS limit, d can be considered to be pure imaginary and the imaginary part of d vanished in the BEC region.

A system of two-component time dependent Ginburg-Landau (TDGL) [1] equations from the Fermion-Boson model (double-channel model), which is one of the microscopic models, has so far been extensively studied, since it can describe a large variety of nonequilibrium dynamics observed in physical systems [2]. In the superfluid atomic Fermi gases near the Feshbach resonance, the strong attractive interaction is realized between Fermion atoms, which can cause the BCS-BEC crossover [3–5].

Though the mathematical framework for the Ginzburg-Landau (GL) theory is simple, it has played an important role in the history of superconductivity research, because it can capture almost every unique feature that the superfluid exhibits macroscopically [6]. To the best of our knowledge even the GL theory has not yet been fully studied in the BCS-BEC crossover regime except for a few pioneering works [7, 8] and recent related ones [9–12] for the single-component Fermion systems (single channel model). Although extensive study has been done in the TDGL equation derived from the single-channel model, little progress has been obtained for a two-component time-dependent Ginzburg-Landau (TDGL) [1]. In this paper we are concerned with the global existence and uniqueness theory for the TDGL equations, i.e., the GL coefficients are generally complex numbers in these systems, for superfluid atomic Fermi-gases near the Feshbach resonance that shows the BCS-BEC crossover [1].

We shall use the following conventional notations throughout the paper. Let L^k_{per} and $H^k_{per}, k=1,2,\cdots$ denote the Hilbert and Sobolev spaces of L-periodic, complex-valued functions endowed with the usual L^2 inner product $(f,g)=\int f\bar{g}\mathrm{d}x$ and norms

$$\|f\|_{L^2}=\sqrt{(f,g)}, \quad \|f\|_{H^k}=\left(\sum_0^k\|\partial^j u/\partial x^j\|\right)^{\frac{1}{2}}.$$

Here $\bar{g}$ denotes the complex conjugate of g. For brevity we write $\|f\|_{L^2}=\|f\|$ and denote the L^p-norm by $\|f\|_p=\left(\int|f|^p\mathrm{d}x\right)^{1/p}$.