JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
Vol. **22**, No. 1, pp. 57-73

J. Part. Diff. Eq.
February 2009

Boundary Homogenization in the Spontaneous Potential Well-Logging

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Received 16 April 2008; Accepted 26 May 2008

Abstract. Motivated by the study on the spontaneous potential well-logging, this paper deals with the homogenization of boundary conditions for a class of elliptic problems with jump interface conditions.

AMS Subject Classifications: 35J25

Chinese Library Classifications: O175.25

Key Words: Jump interface condition; boundary condition with equivalued surface; homogenization of boundary condition; elliptic problem.

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1 Introduction

In petroleum exploitation one often uses various methods of well-logging, among which the spontaneous potential (SP) well-logging is one of the most common and important techniques. Since positive ions and negative ions have different diffusion speeds in a solution, and the grains of mud-stone often absorb positive ions, there is a steady potential difference called the *spontaneous potential difference* on any interface of different formations. These potential differences cause a spontaneous potential field in the earth. After a well has been drilled, one puts a log-tool with a measuring electrode into the well and then measures the SP on the electrode. Raising the tool along the well-bore one gets the corresponding SP curve, as shown in Fig. 1. The SP on the electrode varies with the change of the rock formation, and it shows the osmotic formation clearly (cf. [1–4]).

As usual, we suppose that the formation is symmetric about the well axis and the central plane (see Fig. 2), then the SP field has the same symmetry. Therefore, we consider only the corresponding two-dimensional problem instead of the three-dimensional one. In addition, since the influence of the electric field is very little far apart from the

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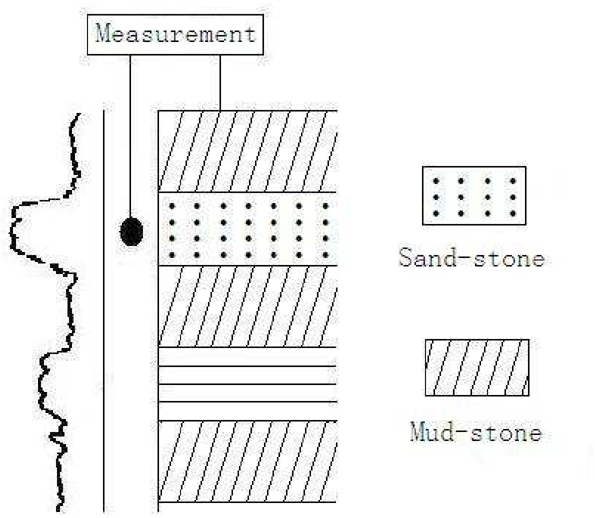

Figure 1: Illustration of measurement for the well-logging.

electrode, we may suppose that the field exists only on a finite but quite large region (cf. [5,6]).

Taking the center of the electrode as the origin and the well axis as the z axis (see Fig. 2), we consider the problem on the domain

$$\Omega \triangleq \{(r,z) | 0 \le r \triangleq \sqrt{x^2+y^2} \le R, 0 \le z \le Z\},$$

where R and Z are suitably large positive numbers. Suppose furthermore that the resistivity of the earth is piecewise constant:

$$Re = \begin{cases} R_m \triangleq R_1 & \text{in } \Omega_m \triangleq \Omega_1, \\ R_s \triangleq R_2 & \text{in } \Omega_s \triangleq \Omega_2, \\ R_{x_0} \triangleq R_3 & \text{in } \Omega_{x_0} \triangleq \Omega_3, \\ R_t \triangleq R_4 & \text{in } \Omega_t \triangleq \Omega_4. \end{cases}$$

In Fig. 2, the shaded part is the area occupied by the log-tool, whose top surface is insulated; Ω_m is the well-bore filled by mud; Ω_s is the enclosing rock; Ω_{x_0} and Ω_t are two parts of the objective formation, which is the main object to be measured. Since the objective formation is usually composed of porous sand-stone, the mud filtrate penetrates into the porous region and changes the resistivity in the domain Ω_{x_0}, which is then called the invaded zone.

If the geometrical structure of the formation, the resistivity in each subdomain and the SP difference on each interface are all known, as a direct problem, the spontaneous potential $u(r,z)$ satisfies the following quasi-harmonic equation in each subdomain Ω_i $(1 \le i \le 4)$:

$$Lu = 0,$$