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SOME LIOUVILLE TYPE THEOREMS FOR THE P-SUB-LAPLACIAN ON THE GROUP OF HEISENBERG TYPE*

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Abstract In this paper we prove some Liouville type results for the p-sub-Laplacian on the group of Heisenberg type. A strong maximum principle and a Hopf type principle concerning p-sub-Laplacian are established.

Key Words p-sub-Laplacian; Liouville type theorems; group of Heisenberg type; Strong maximum principle.

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1. Introduction

Nonexistence results for the semilinear elliptic and subelliptic inequalities have been studied, see [1–4]. Birindelli and Demengel [5] proved some Liouville results concerning the p-Laplacian on the Euclidean space by using the properties of viscosity solution and local test function. In [6], Vazquez established a strong maximum principle and a Hopf type principle for the partial differential inequalities concerning p-Laplacian. The results were extended to a wide class of inequalities in [7]. The Hopf type principles for the sub-Laplacian on the Heisenberg group and on the group of Heisenberg type have been proved, see [8] and [9], respectively. We note that the strong maximum principle

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and the Hopf type principle concerning p-sub-Laplacian on the Heisenberg group or the group of Heisenberg type have not been seen.

The purpose of the present paper is to consider the p-sub-Laplacian on the group of Heisenberg type and give some Liouville type theorems. Our approach is based on the adaptation of ideas in [5] and [7].

We denote by G the group of Heisenberg type and Q the homogeneous dimension of G. The p-sub-Laplacian on G is given by

$$L_p u = \sum_{j=1}^{m} X_j(|Xu|^{p-2} X_j u),$$

where u is a differentiable function.

Let $\rho(\xi, \eta)$ be the natural distance in G. In the following, we denote by $\rho(\xi)$ the distance from ξ to e (the group identity),

$$\rho(\xi) = (|x(\xi)|^4 + 16|y(\xi)|^2)^{\frac{1}{4}}$$

and by $W^{1,p}_{loc}(G)$ the localization of Sobolev type space $W^{1,p}(G)$, see [10].

Our main results are the following.

Theorem 1.1 *Suppose that $Q > p > 1$. Let $u \in W^{1,p}_{loc}(G) \cap C(G)$ be a nonnegative weak solution of*

$$L_p u + h u^q \leq 0 \quad \text{in} \quad G, \tag{1.1}$$

with the nonnegative function h satisfying

$$h(\xi) = c\rho^{\gamma}, \quad \text{for} \quad \rho \quad \text{large}, \quad c > 0 \quad \text{and} \quad \gamma > -p.$$

If $0 < q \leq \frac{(Q+\gamma)(p-1)}{Q-p}$, then $u \equiv 0$.

Theorem 1.2 *Let $Q \leq p$. If $u \in W^{1,p}_{loc}(G) \cap C(G)$ is bounded below and a weak solution of*

$$-L_p u \geq 0 \quad \text{in} \quad G, \tag{1.2}$$

then u is constant.

This paper is organized as follows. In Section 2 we collect some tools on the group of Heisenberg type which will play a role in the following sections. In Section 3 we provide some lemmas, including a strong maximum principle, a Hopf type principle and a Hadamard type inequality. They have independent interest. Section 4 is devoted to the proof of theorems. In Section 5 we give an explicit solution for some partial differential equation.