J. Partial Diff. Eqs. **21**(2008), 45–58

Vol.21 No.1

# HYPERBOLIC-PARABOLIC CHEMOTAXIS SYSTEM WITH NONLINEAR PRODUCT TERMS*

Chen Hua and Wu Shaohua
(School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China)
(Email: chenhua@whu.edu.cn)

Dedicated to the 70th birthday of Professor Li Tatsien

(Received July. 25, 2007)

**Abstract** We prove the local existence and uniqueness of week solution of the hyperbolic-parabolic Chemotaxis system with some nonlinear product terms. For one dimensional case, we prove also the global existence and uniqueness of the solution for the problem.

**Key Words** Hyperbolic-parabolic system; Chemotaxis; external signal.
**2000 MR Subject Classification** 35K50, 35M10, 35R25, 92C45.
**Chinese Library Classification** O175.29.

## 1. Introduction

Let $u(x,t)$ and $v(x,t)$ represent the population of an organism and an external signal at place $x \in \Omega \subset R^N$ and time $t$ respectively, in general speaking, the external signal is produced by the individuals and decays, which is described by a nonlinear function $g(v,u)$. Under the spatial spread of the external signal is driven by diffusion, the full system for $u$ and $v$ reads (see [1-3])

$$u_t = \nabla(d\nabla u - \chi(v)\nabla v \cdot u), \tag{1}$$

$$v_t = d\Delta v + g(v,u). \tag{2}$$

In the case of that the external stimulus were based on the light (or the electromagnetic wave), H. Chen and S. Wu [4] studied following hyperbolic-parabolic type chemotaxis system:

$$u_t = \nabla(d\nabla u - \chi(v)\nabla v \cdot u), \tag{3}$$

$$v_{tt} = d\Delta v + g(v,u), \tag{4}$$

*Research supported by NSFC (No.10631020).

where $v$ represents the potential function of the external signal, for example, if the external signal is the electromagnetic field, then $v$ would be voltage (in this case $\nabla v$ denotes the electromagnetic field).

The result of [4] gives the existence and uniqueness of the solution for the system (3)-(4) with Neumann boundary value condition on a smoothly bounded open domain $\Omega$ and $g(v,u) = -v + f(u)$. In this paper, we shall study the case for more general nonlinear term $g(v,u)$.

Throughout this article, we assume that we can choose a constant $\sigma$, satisfying

$$1 < \sigma < 2, \tag{5}$$

$$N < 2\sigma < N + 2, \tag{6}$$

$$\sigma - 1 \geq \frac{N}{4}, \tag{7}$$

where $1 \leq N \leq 3$ are space dimensions.

It is easy to check that there exists some constant $\sigma$ such that the three conditions above can be simultaneously satisfied in the cases of $1 \leq N \leq 3$. In fact, we can choose $\sigma = \frac{5}{4}$ for $N = 1$, $\sigma = \frac{13}{8}$ for $N = 2$ and $\sigma = \frac{15}{8}$ for $N = 3$.

Next, we define

$$\begin{aligned}
X_{t_0} &= C([0,t_0], H^\sigma(\Omega) \cap \{\frac{\partial u}{\partial n} = 0 \ \ on \ \ \partial\Omega\}),\\
X_\infty &= C([0,+\infty), H^\sigma(\Omega) \cap \{\frac{\partial u}{\partial n} = 0 \ \ on \ \ \partial\Omega\}),\\
Y_{t_0} &= C([0,t_0], H^2(\Omega) \cap \{\frac{\partial v}{\partial n} = 0 \ \ on \ \ \partial\Omega\}) \cap C^1([0,t_0], H^1(\Omega)),\\
Y_\infty &= C([0,+\infty), H^2(\Omega) \cap \{\frac{\partial v}{\partial n} = 0 \ \ on \ \ \partial\Omega\}) \cap C^1([0,+\infty), H^1(\Omega)),
\end{aligned}$$

and

$$Z_{t_0} = C^1([0,t_0], L^2(\Omega)), \qquad Z_\infty = C^1([0,\infty), L^2(\Omega))),$$

$$W_{t_0} = C^2([0,t_0], L^2(\Omega)), \qquad W_\infty = C^2([0,\infty), L^2(\Omega)).$$

## 2. Local Existence and Uniqueness for $g(u,v) = \alpha uv$

In this section we consider following system in which the nonlinear function $g(u,v)$ is a product term:

$$\begin{cases}
u_t = \nabla(\nabla u - \chi u \nabla v), \quad \text{in} \quad \Omega \times (0,T),\\
v_{tt} = \Delta v + \alpha uv, \quad \text{in} \quad \Omega \times (0,T),\\
\dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0, \quad \text{on} \quad \partial\Omega \times (0,T),\\
u(0,\cdot) = u_0, \quad v(0,\cdot) = \varphi, \quad v_t(0,\cdot) = \psi, \quad \text{in} \quad \Omega,
\end{cases} \tag{8}$$