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THE UNCONDITIONAL STABILITY OF PARALLEL DIFFERENCE SCHEMES WITH SECOND ORDER CONVERGENCE FOR NONLINEAR PARABOLIC SYSTEM*

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Abstract For solving nonlinear parabolic equation on massive parallel computers, the construction of parallel difference schemes with simple design, high parallelism and unconditional stability and second order global accuracy in space, has long been desired. In the present work, a new kind of general parallel difference schemes for the nonlinear parabolic system is proposed. The general parallel difference schemes include, among others, two new parallel schemes. In one of them, to obtain the interface values on the interface of sub-domains an explicit scheme of Jacobian type is employed, and then the fully implicit scheme is used in the sub-domains. Here, in the explicit scheme of Jacobian type, the values at the points being adjacent to the interface points are taken as the linear combination of values of previous two time layers at the adjoining points of the inner interface. For the construction of another new parallel difference scheme, the main procedure is as follows. Firstly the linear combination of values of previous two time layers at the interface points among the sub-domains is used as the (Dirichlet) boundary condition for solving the sub-domain problems. Then the values in the sub-domains are calculated by the fully implicit scheme. Finally the interface values are computed by the fully implicit scheme, and in fact these calculations of the last step are explicit since the values adjacent to the interface points have been obtained in the previous step. The existence, uniqueness, unconditional stability and the second order accuracy of the discrete vector solutions for the parallel difference schemes are proved. Numerical results are presented to examine the stability, accuracy and parallelism of the parallel schemes.

Key Words Parallel difference scheme; nonlinear parabolic system; unconditional stability; second order convergence.

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1. Introduction

There is rich literature on the parallel difference schemes for the parabolic equation (see [1-5] [6-10]).Explicit schemes are naturally parallel and also easy to implement, but they usually require small time steps because of stability constraints. Implicit schemes are necessary for finding steady state solution or computing slowly unsteady problems where one needs to march with large time steps. However, implicit schemes are not inherently parallel. The parallel schemes in [4, 5]and [9, 10] use the explicit scheme and the implicit scheme alternately in the time and space direction, which can implement the parallel computation and are unconditionally stable. These schemes involve three time layers in essence, and have been extended to semi-linear parabolic equation in [7]. Their truncation error is $O(\tau + h)$, where τ and h are the time and space step respectively. Furthermore they suffer from the defect that the truncation error of the alternating schemes can not be eliminated for the general nonlinear parabolic problem. For clarity we recall the definition of the unconditional stability. Let $r = \frac{\tau}{h^2}$. If a difference scheme is stable for all small τ and h satisfying $r \le \Lambda$ with Λ being any fixed positive constant, then we call it is unconditionally stable. For the heat equation $u_t = u_{xx}$ the fully implicit scheme is unconditional stable, while the fully explicit scheme is not unconditional stable since the constant Λ cannot be taken larger than $\frac{1}{2}$ in this case.

A natural way to solve partial differential equations in parallel is to divide the domain over which the problem is defined into sub-domains, and solves the sub-domain problems in parallel. The major difficulties with such procedures involve defining values on the sub-domain boundaries and piecing the solutions together into a reasonable approximation to the true solution. Once the interface values are available, the global problem is fully decoupled and thus computed in parallel. A parallel scheme was proposed in [1], where instead of using the same spacing h as for the interior points where the implicit scheme is applied, a larger spacing H_D is used at each interface point where the explicit scheme is applied. There are also some other schemes with domain decomposition in [2, 3]. These schemes are conditionally stable. Since unconditional stable schemes are desired in many applications, some unconditional stable schemes were proposed in [8], which firstly take the values of previous time step as the boundary condition, and then solve the sub-domain problems in parallel, and finally update the interface values between sub-domains by the implicit scheme. These schemes can be easily implemented in the parallel computer, but their convergence order is only one order. In order to improve the convergence order, the parallel iterative difference schemes based on interface correction for parabolic equation were proposed in [6], which are complicated in requiring the correction of the interface values in the process of iteration of solving sub-domain problems.

In this paper we propose a new kind of general parallel difference schemes for the nonlinear parabolic problem. The resulting schemes are of second order global accuracy in space and unconditionally stable as well. The general parallel difference schemes