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ON A HIGH ORDER SPIN WAVE SYSTEM WITH A NONLINEAR FREE TERM

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Dedicated to Professor Jiang Lishang on the occasion of his 70th birthday

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Abstract In this paper, we deduct a new spin wave model in lattices, which is a nonlinear high order degenerate parabolic system with a nonlinear free term. In a further theoretical study, by using a parameter ϵ approximation, the existence of a weak solution has been obtained.

Key Words Spin wave; high order system; degenerate parabolic equations; existence of solution.

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1. Modelling and Problem

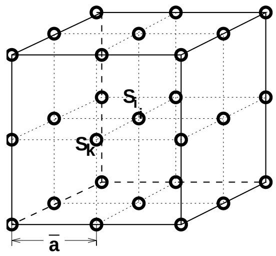
In Solid State Physics an important concept is that of collective excitations ([1-3]). Collective excitations are the low-lying excited states of systems where a strong coupling between particles is present. Their nature and origin can be varied and depends on the system and interaction considered. For example, the most notable of these are the lattice vibrations of a crystalline structure, which, when properly quantized are called phonons. Other collective excitations of particular importance, and that we will consider here, are spin wave excitations. These low-lying excitations occur in ferromagnets and correspond to the oscillations of the electron-spin-density fluctuations. To be more specific, in a ferromagnet below the Curie temperature, due to the exchange interaction, the magnetic moments associated with each lattice site are lined up so that they all statistically point in the same direction. This corresponds to the ground state of the system. Spin waves are the excited eigenstates of the system Hamiltonian which, in a classical sense, correspond to the propagation of spin deviations from the original direction.

Spin wave excitations in ferromagnetic lattices can be characterized by a spin-exchange Hamiltonian which is invariant under lattice translation. That is, after calculating the quantum equation of motion with the spin Hamiltonian, spin vectors $\boldsymbol{S}$

should satisfy the following relationship, (see [3-5]),

$$\frac{\hbar}{2}\frac{\partial \boldsymbol{S}_i}{\partial t} = \sum_{k\neq i}\frac{A}{2}[\boldsymbol{S}_i, \boldsymbol{S}_k] + [\boldsymbol{S}_i, \boldsymbol{h}],$$

where i points to the ith atom. The summing index k points to a neighbor atom of the ith site; A is the exchange integral; $\hbar$ is Planck constant; the vector $\boldsymbol{h}$ is a given function which may depend on $\boldsymbol{S}_i$. The square brackets $[\cdot,\cdot]$ denote the commutator of the two vectors which describes the effect between the atoms.



simple cubic lattice

A kind of materials, such as $\alpha-$Fe (see [3-5]),with a ferromagnetic property is a simple cubic lattice with lattice constant $\bar{a}$. Suppose that a smooth function $\boldsymbol{S}$ values $\boldsymbol{S}_i$ at the ith atom, i.e. $\boldsymbol{S}$ is continuous and smooth enough and $\boldsymbol{S}(x) = \boldsymbol{S}_i(x)$, then $\boldsymbol{S}_k(x) = \boldsymbol{S}(x \pm \bar{a})$, where $\boldsymbol{S}_k$ correspond to those atoms adjacent the ith atom, $x = (x_1, x_2, x_3)$. Those $\boldsymbol{S}_k$s can be expanded and expressed by $\boldsymbol{S}$ as follows:

$$\boldsymbol{S}(x_1 \pm \bar{a}, x_2, x_3) = \boldsymbol{S}(x) \pm \frac{\partial \boldsymbol{S}}{\partial x_1}(x) + \frac{1}{2!}\frac{\partial^2 \boldsymbol{S}}{\partial x_1^2}(x) \pm \frac{1}{3!}\frac{\partial^3 \boldsymbol{S}}{\partial x_1^3}(x) + \cdots,$$

the same way for $\boldsymbol{S}(x_1, x_2\pm\bar{a}, x_3)$, $\boldsymbol{S}(x_1, x_2, x_3\pm\bar{a})$, $\boldsymbol{S}(x_1\pm\bar{a}, x_2\pm\bar{a}, x_3)$, $\cdots$ and so on.

Sum all $\boldsymbol{S}_k$ around x. By the symmetry of the lattices, all the items with odd differential degree are eliminated. Therefore, we have

$$\frac{A}{2}\sum_k \boldsymbol{S}_k(x) = \sum_{m=0}^{\infty} \widetilde{\triangle}^m \boldsymbol{S}(x),$$

where

$$\widetilde{\triangle}^m = \sum_{|\alpha|=m} a_\alpha D_x^{2\alpha}, \qquad (m = 1, 2, \cdots) \tag{1.1}$$

are elliptic operators. α is a $N-$tuple index, i.e. $\alpha=(\alpha_1, \cdots, \alpha_N)$, with $\{\alpha_i\}_{i=1,\cdots,N}$ are non-negative integrals, $|\alpha|=\sum_{j=1}^N \alpha_j = m$, $m = 1, 2, \cdots, M$. For any α defined above, a_α is a positive constant depending on $\bar{a}$. Then

$$\boldsymbol{S}_t = \boldsymbol{S}\times \sum_{m=0}^{\infty} \widetilde{\triangle}^m \boldsymbol{S} + \boldsymbol{S}\times\boldsymbol{h}, \tag{1.2}$$