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GLOBAL SOLUTION AND ITS LONG TIME BEHAVIOR FOR THE GENERALIZED LONG-SHORT WAVE EQUATIONS*

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Abstract The long time behavior of the solutions of the generalized long-short wave equations with dissipation term is studied. The existence of global attractor of the initial periodic boundary value is proved by means of a uniform a priori estimate for time. And also the dimensions of the global attractor are estimated.
Key Words The generalized long-short wave equations; a uniform priori estimate; global attractor; Hausdorff dimension; fractal dimension.
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1. Introduction

The long-short wave equations,

$$iS_t + S_{xx} - LS = 0$$

$$L_t + |S|_x^2 = 0$$

are first derived by Djordjevic and Redekopp in [1], where S is the envelope of the short wave, L is an amplitude of long wave and is real. This system describes the resonance interaction between the long wave and the short wave. Furthermore, Funakoshi and Oikawa derived the following system [2]

$$iS_t + S_{xx} - LS = 0$$

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$$L_t + lHL_{xx} = m|S|_x^2$$

where l, m are physical constants and H represents the Hilbert transform [3] defined by

$$Hu(x,t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u(y,t)}{y-x} dy$$

This system describes the interaction of two fluid interfaces under the setting of deep flow in [4]. Guo Bo-ling obtains the existence of global solution for long-short wave equations in [5]. The approximation inertial manifolds for LS type equations have been studied in [6].

In this paper, we consider the global solution and its long time behavior for the following initial periodic boundary value problem of the generalized LS type with dissipative term equations:

$$iu_t + u_{xx} - nu + i\alpha u + g(|u|^2)u + h_1(x) = i\beta u_{xx} \tag{1}$$

$$n_t + |u|_x^2 + lHn_{xx} + \delta n + f(|u|^2) + h_2(x) = \gamma n_{xx} \tag{2}$$

$$u(x,0) = u_0(x), \quad n(x,0) = n_0(x), \quad x \in \Omega = (-D, D), D > 0 \tag{3}$$

$$u(x-D,t) = u(x+D,t), \quad n(x-D,t) = n(x+D,t) \tag{4}$$

where $\alpha, \beta, \gamma, \delta$ are positive constants, and $l = 0$ is constant. $u(x,t)$ is an unknown complex valued vector, $n(x,t)$ is an unknown real valued function, $h_1(x)$and $h_2(x)$ are given in $L^2(\Omega)$, $g(s)$ and $f(s)(0 \leq s < \infty)$ are smooth real valued functions which satisfy

$$|g(s)| \leq c_1 s^{2-\sigma} + c_2, \quad s \geq 0 \tag{5}$$

$$|f(s)| \leq c_3 s^{\frac{3}{2}-\nu} + c_4. \quad s \geq 0 \tag{6}$$

where σ, ν and c_i are positive constants. By means of a uniform a priori estimates for time, and Temam's [7] methods, we obtain the above results.

Throughout this paper, the C will be used to indicate generic constants and dependent of data$(\alpha, \beta, \gamma, \delta, f, g, h_1, h_2, R)$. We denote by $\|\cdot\|$ the norm of $H = L^2_{per}(\Omega)$ (real or complex space) with the corresponding inner product $(\cdot,\cdot)$, denote by $\|\cdot\|_p$ the norm of $L^p_{per}(\Omega)$ for all $1 \leq p \leq \infty (\|\cdot\|_2 = \|\cdot\|)$, and $\|\cdot\|_x$ the norm of any Banach space X.

2. Uniform a Priori Estimates in Time

Lemma 1 *If $u_0 \in L^2(\Omega)$, and $h_1(x), h_2(x) \in L^2(\Omega)$, then for the solution (u,n) of problem (1)-(4) we have*

$$\|u\|^2 \leq e^{-\alpha t}\|u_0\|^2 + \frac{1}{\alpha^2}(1 - e^{-\alpha t})\|h_1\|^2. \tag{7}$$

Furthermore

$$\overline{\lim}_{t\to\infty}\|u(\cdot,t)\|^2 \leq \frac{1}{\alpha^2}\|h_1\|^2 \equiv E_0 \tag{8}$$