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THE GLOBAL WELLPOSEDNESS AND SCATTERING OF THE GENERALIZED DAVEY-STEWARTSON EQUATION

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Abstract We discuss the solution of the Cauchy problem of the generalized Davey-Stewartson equation. When the initial value is small enough, we obtain the global wellposedness of the solution and scattering.
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1. Introduction

In this paper we will prove the global wellposedess and scattering result for the Cauchy problem of the generalized Davey-Stewartson equation when the datum is small enough.In [1], Wang Baoxiang, Guo Boling studied the generalized Davey-Stewartson equation,

$$iu_t + Au = \lambda_1|u|^{p_1}u + \lambda_2|u|^{p_2}u + \mu E(|u|^2)u, \tag{1.1}$$

where $u(t,x)(x=(x_1,x_2,...,x_n))$is a complex function of $(t,x)\in R_+\times R^n$. $\lambda_1,\lambda_2,\mu\in C$,

$$A := \sum_{1\le i,j\le n} a_{ij}\frac{\partial^2}{\partial x_i\partial x_j},$$

$$E(\varphi) = \mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}\varphi. \tag{1.2}$$

In the above, $\mathcal{F}(\mathcal{F}^{-1})$ denotes Fourier (converse) transform, $(a_{ij}),(b_{ij})$ are real invertible matrices satisfying

$$\Big|\sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j\Big| \ge C|\xi|^2, \forall\xi\in R^n, \tag{1.3}$$

In this paper, we will study the initial value problem of the generalized Davey-Stewartson equation with the form :

$$iu_t + Au = \lambda|u|^{2q-2}u + \mu E(|u|^q)|u|^{q-2}u, \tag{1.4}$$

$$u(0,x) = u_0(x), \tag{1.5}$$

where $\lambda, \mu \in$ C, $A, E(\varphi)$are defined in (1.2), respectively.

For any $4/n \le p < \infty$ and $r \in [2,\infty)$,we denote:

$$s(p) = \frac{n}{2} - \frac{2}{p}, \quad \frac{2}{\gamma(r)} = n(\frac{1}{2} - \frac{1}{r}), \quad r(p) = \frac{2n(2+p)}{n(2+p)-4}, \tag{1.6}$$

$$\alpha(n) = \begin{cases} \dfrac{2n}{n-2}, & n > 2 \\ \infty, & n = 2 \end{cases}. \tag{1.7}$$

Our main result is as follows:

Theorem 1.1 *Suppose* $n \ge 2, 2 \le q < \infty, s(2q-2) = \dfrac{n}{2} - \dfrac{1}{q-1}$,*and there exists* $\delta_1 > 0$, *such that, when* $\|u_0\|_{H^{2q-2}} \le \delta_1$, *(1.4),(1.5) has a unique solution satisfying*

$$u \in C\left(0,\infty; H^{s(2q-2)}\right) \bigcap \bigcap_{2<r<\alpha(n)} L^{\gamma(r)}\left(0,\infty; B^{s(2q-2)}_{r,2}\right).$$

Theorem 1.2 *Suppose* $n \ge 2, 2 \le q < \infty, s(2q-2) = \dfrac{n}{2} - \dfrac{1}{q-1}$,*and there exists* $\delta_1 > 0$ *such that, when* $\|u_0\|_{H^{2q-2}} \le \delta_1$,*the solution of (1.4)(1.5) has scattering.*

The proof of Theorem 1.2 is omitted.

Let S(t) be a semi-group generated by $i\frac{\partial}{\partial t} + A$. From [2] we can obtain the time-space Strichartz estimate:

$$\|S(t)f\|_{L^{\gamma(r)}(-\infty,\infty;\dot{B}^s_{r,2})} \le \|f\|_{\dot{H}^s}, \tag{1.8}$$

$$\left\| \int_0^t S(t-\tau)f(\tau)d\tau \right\|_{L^{\gamma(r)}(0,T;\dot{B}^s_{r,2})} \le C\|f\|_{L^{\gamma(q)'}(0,T;\dot{B}^s_{q',2})}, \tag{1.9}$$

where $q, r \in [2,\alpha(n)), 0 < T \le \infty$, and C is independent of T. If $f = \sum_{i=1}^I f_i, \quad r, q_i \in [2,\alpha(n)), i = 1,2,...,I$, from (1.9) we get:

$$\left\| \int_0^t S(t-\tau)f(\tau)d\tau \right\|_{L^{\gamma(r)}(0,T;\dot{B}^s_{r,2})} \le C\sum_{i=1}^I \|f_i\|_{L^{\gamma(q_i)'}(0,T;\dot{B}^s_{q_i',2})}. \tag{1.10}$$

2. The Nonlinear Estimates

Lemma 2.1([1]) $\forall 1 < p < \infty$, *we get:*

$$\rho(\xi) =: \frac{\xi_1^2}{\sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j} \in \mathcal{M}_p.$$

where (b_{ij}) *satisfies (1.3),* $\mathcal{M}_p$ *denotes multiplier space.*