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EXPONENTIAL ATTRACTOR FOR A CLASS OF NONCLASSICAL DIFFUSION EQUATION*

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Abstract In this paper, we consider the asymptotic behavior of solutions for a class of nonclassical diffusion equation. We show the squeezing property and the existence of exponential attractor for this equation. We also make the estimates on its fractal dimension and exponential attraction.

Key Words Nonclassical; diffusion equation; squeezing property; exponential attractor

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1. Introduction

The nonclassical diffusion equations

$$u_t - \nu\Delta u_t - \sum_{i=1}^{n}[\sigma(u_{x_i})]_{x_i} + g(u) = f(x,t) \tag{1.1}$$

arise in many different areas of mathematics and physics. They have been used, for instance, to model thermodynamics processes[1], [2], fluid flow in fissured rock [3],consolidation of clay [4], and shear in second order fluids [5-7]. For the physical interpretation of "$\nu\Delta u_t$", we refer to [1-3]. The equations of (1.1) with a one time derivative appearing in the highest order term are called pseudo-parabolic or Sobolev-Galpern equations [8-12]. Aifantis [13] proposed a general frame for establishing the equations. The existence and uniqueness, and regularity of solutions for the nonclassical diffusion equations have been investigated by many authors, such as Showalter [14], Davis [15],

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Quarteroni [16], Karch [17], Shi *et al* [18], Liu and Wang [19], Liu,Wan and Lu [20], Li *et.al* [21] and their references therein.

In this paper,we consider the following initial boundary value problem of the non-classical diffusion equation:

$$u_t - \nu\Delta u_t - \lambda\Delta u + g(u) = f(x), (x,t) \in \Omega \times R^+, \tag{1.2}$$
$$u(x,0) = u_0(x), \qquad x \in \Omega, \tag{1.3}$$
$$u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times R^+. \tag{1.4}$$

where λ is a positive constant, $g : R \to R$ is a smooth function, and $f(x) \in L^2(\Omega), \Omega \subset R^n$ is a smooth bounded open set, $\partial\Omega$ is the boundary of Ω. The existence of the compact global attractor in $H_0^1(\Omega)$ for the equation (1.2) has been established by Li *et al* [21]. Our aim of this paper is to show the existence of finite dimensional exponential attractor for this equation.

The outline of this paper is as follows: in Section 2, we state some basic results on the existence of exponential attractors and recall some known results concerning the existence and uniqueness of solutions. Section 3 contains our main results; we first establish the Lipschitz continuity of the dynamical system $S(t)$ associated with Eq.(1.2), then we prove that the semigroup $S(t)$ satisfies the squeezing property and deduce the existence of the exponential attractor.

Throughout this paper, we denote by $\|\cdot\|$ the norm of $H = L^2(\Omega)$ with the usual inner product $(\cdot,\cdot)$. We also use $\|\cdot\|_p$ for the norm of $L^p(\Omega)$ for $1 \le p \le \infty (\|\cdot\|_2 = \|\cdot\|)$. Generally, $\|\cdot\|_X$ denotes the norm of Banach space X.

For convenience, we put $\nu \equiv 1$ in (1.2). In the sequel, we always assume that g satisfies conditions:

$(G_1) g \in C^1(R), \exists \mu \in R$, such that $\underline{\lim}_{|s|\to\infty} \dfrac{g(s)}{s} \ge \mu$;

$(G_2) \exists c, \gamma \ge 0$, such that

$$|g'(s)| \le c(1 + |s|^\gamma),$$

where $0 \le \gamma < \infty$(as $n = 1, 2), \gamma \le \dfrac{2n}{n-2}$(as $n \ge 3$).

2. Preliminaries

Let $D(A), V$ be two Hilbert spaces, $D(A)$ be dense in V and compactly imbedded into V.

We study

$$\frac{du}{dt} + Au + g(u) = f(x), \quad t > 0, \quad x \in \Omega. \tag{2.1}$$
$$u(0) = u_0, \quad x \in \Omega \tag{2.2}$$
$$u\mid_{\partial\Omega} = 0. \tag{2.3}$$