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THE CAUCHY PROBLEM FOR A CLASS OF COUPLED SYSTEMS CONTAINING A CONVOLUTION OPERATOR*

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Abstract Some results on the invariant regions ,existence and uniqueness of solutions to a class of integrodifferential systems are established. Applying these results to integrodifferential systems with a small parameter $\varepsilon > 0$, we obtain,in particular, some estimates of solutions uniform in $\varepsilon > 0$.
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1. Introduction

In this paper, we are concerned with the following integrodifferential systems

$$\begin{cases} u_t - J * u + u + f(u,v) = 0, & x \in R^n, \quad t > 0, \\ v_t - D\Delta v + g(u,v) = 0, & x \in R^n, \quad t > 0 \end{cases} \tag{1.1}$$

and

$$\begin{cases} u_t - J * u + u + f(u,v) = 0, & x \in R^n, \quad t > 0, \\ v_t - K * v + g(u,v) = 0, & x \in R^n, \quad t > 0, \end{cases} \tag{1.2}$$

where D is a positive constant and $J * u$ and $K * v$ are convolution operators:

$$(J * u)(x,t) = \int_{R^n} J(y)u(x-y,t)dy,$$

$$(K * v)(x,t) = \int_{R^n} K(y)v(x-y,t)dy,$$

with $J(x), K(x)$ satisfying:

$$\begin{aligned} &J(x), K(x) \in C^2(R^n), \quad J(x) \geq 0, \quad K(x) \geq 0, \\ &\int_{R^n} J(x)dx = \int_{R^n} K(x)dx = 1, \quad J(x) = J(-x), K(x) = K(-x). \end{aligned} \tag{A}$$

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The corresponding initial conditions are

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in R^n. \tag{1.3}$$

A lot of phenomena in chemistry,physics and biology can be described by this kind of equations mathematically. It should be pointed out that the substantial difference between (1.1), (1.2) and the parabolic system

$$\begin{cases} u_t = \alpha\Delta u - f(u,v), & x \in R^n, t > 0, \\ v_t = \beta\Delta v - g(u,v), & x \in R^n, t > 0, \end{cases} \tag{1.4}$$

where α, $\beta > 0$, is that Δu and Δv in (1.4) only depend on the values of u and v locally in space, while $J * u$ and $K * v$,as integrals over R^n, are dependent on the values of u and v in the whole space R^n. This means that (1.1) and (1.2) are the nonlocal analog of (1.4).

P. Bates, P. C. Fife, X. F. Ren and X. F. Wang ([1]) have discussed the existence and uniqueness of travelling waves for the following integrodifferential equation

$$u_t - J * u + u + f(u) = 0, \qquad x \in R^n, \tag{1.5}$$

and the asymptotic behavior of solutions for some perturbed equation related to (1.5). For other integrodifferential equations, Ermentrout, Mcleod([2]) and Orlandi, Triolo([3]) have been concerned with the equation

$$u_t = \tanh\{\beta(J * u + h)\} - u,$$

where $\beta > 1$, h are constants.

It has been noticed in [1] and [4] that as the nonlocal analog of the parabolic equation

$$u_t = D\Delta u - f(u), \tag{1.6}$$

the equation (1.5) shares some properties with (1.6), such as a form of maximum principle.

In this paper, we establish the general theory including the existence and uniqueness of bounded solutions for (1.1), (1.3) and (1.2), (1.3). First of all, we prove in Section 2 some theorems on the invariant regions for the systems (1.1) and(1.2) under some conditions on the nonlinear terms $f(u,v)$ and $g(u,v)$. On the basis of these results, the existence theory for (1.1) and (1.2) is established in Section 3. Finally, we apply these results to the integrodifferential systems with a small parameter $\varepsilon > 0$

$$\begin{cases} \varepsilon u_t - J_\varepsilon * u + u + f(u,v) = 0, \\ v_t - D\Delta v + g(u,v) = 0, \end{cases} \tag*{$(1.1)_\varepsilon$}$$