J. Partial Diff. Eqs. **16**(2003), 135–147
 Vol.16 No.2

GLOBAL COUPLING OF AN INTERFACE PROBLEM IN AN ACTIVATOR-INHIBITOR SYSTEM*

YoonMee Ham
(Department of Mathematics, Kyonggi University, Suwon, 442-760 Korea)
(E-mail:ymham@kuic.kyonggi.ac.kr)
(Received Mar. 19, 2002; revised Oct. 9, 2002)

Abstract An activator-inhibitor reaction system with global coupling was introduced in [1]. The authors showed that global coupling suppresses the breathing motion and enhances the propagation of the localized solution. The collision between two traveling waves for a sufficiently strong global coupling is discussed in [2]. If the width of layers is infinitesimally thin, the equation of motion for a pair of the interfaces is derived. We shall study the dynamics of interfaces in the free boundary problem with global coupling and with a strong global coupling.
Key Words activator-inhibitor; van der Pol equation; global coupling; free boundary problem; Hopf bifurcation.
2000 MR Subject Classification 35R35, 35B32, 35B25, 35K22, 35K57.
Chinese Library Classification O175.26, O175.29, O175.8.

1. Introduction and Interfacial Equations

In this paper, we are concerned with a free boundary problem defined on the band-shaped domain:

$$\begin{cases} v_t + Av = H(x-s) - H(x-\eta), \quad (x,t) \in \Omega^+(t) \cup \Omega^-(t), \\ s'(t) = C((v(s(t)),t);a(t)), \ t>0, \\ \eta'(t) = -C((v(\eta(t),t);a(t)), \ t>0, \\ v(x,0) = v_0(x), \ s(0)=s_0, \ \eta(0)=\eta_0 \end{cases} \tag{1}$$

where $\Omega^+(t) = \{(x,t) : s(t) < x < \eta(t), t > 0\}$ and $\Omega^-(t) = \{(x,t) : -L/2 < x < s(t),\ \eta(t) < x < L/2, t > 0\}$. A differential operator A is represented by $Av = -v_{xx} + (b+1)\,v$ together with the Neumann boundary condition $v_x(-L/2) = 0 = v_x(L/2)$. A function C is a velocity of the interfaces s and η and, assume to be a continuously

*The present work was supported by the Research Foundation at Kyonggi University in 2001.

differentiable function on an interval I. The origin of this problem is the activator-inhibitor system with Bonhoeffer-van der Pol dynamics ([3–6])

$$\begin{cases} \varepsilon\sigma u_t - \varepsilon^2 u_{xx} = f(u,v), \\ v_t - Dv_{xx} = u - bv, \quad t>0,\ x\in(-L/2, L/2), \end{cases} \tag{2}$$

where $f(u,v) = -u + \theta(u-a) - v$ with $\theta(x)=1$ for $x>0$ and $\theta(x)=-1$ for $x<0$. (If f is a cubic form, the linear approximation of this nonlinear dynamic may be obtained as [7]). The constants σ and b are positive and chosen such that the system is excitable and that a localized stable pulse solution exists in [8]. The parameter ε is a measure of the width of the interface. For $a=0$ (without global coupling) and for small ε, it is known that for large value of σ this system admits a stable stationary solution. For the decreasing value of σ, the stationary solutions loose the stability and produce a kind of periodic oscillation in the location of the internal layers ([9–11]). These periodic solutions are called *breathers* or *breathing solutions*. The first of these instabilities leads to breathing of a stationary solution was shown by Koga and Kuramoto [12].

Recently, Krischer and Mikhailov [1] introduced global coupling which plays an important role in stabilizing interface and preventing their blow-up and the excitation threshold a depends on the total activator concentration in the medium such as

$$a = \alpha\Big(\int_{-L/2}^{L/2}(u+v)dx - S_0\Big). \tag{3}$$

The positive constant S_0 is the value of the integral $\int_{-L/2}^{L/2}(u+v)dx$ for the spatial distribution corresponding to a stationary spot; the coefficient α characterizes the intensity of global coupling. For sufficiently large α, (3) simply becomes $a=0$ and $\int_{-L/2}^{L/2}(u+v)dx = S_0$. In the limit $\varepsilon\to 0$, this condition implies that a conservation of the area of a localized domain (width of interfaces) and hence the breathing solution is automatically prohibited. Krischer and Mikhailov [1] proved the bifurcation from a motionless pulse to a propagating pulse becomes supercritical and Ohta [6] investigated that the reflection of pulses in a reaction-diffusion system by a singular perturbation method in higher dimension extending the interface dynamics.

In this paper, we shall study the dynamics of interfaces in the interface problem (1) for the case of global coupling and the case of a strong global coupling. We apply similar techniques in which was used in [7, 13] to examine Hopf bifurcations as the global coupling intensity varies. The authors proved the well posedness of solutions for the single free boundary problem applying the semigroup theory using domains of fractional powers of A with the domain

$$D(A) = \{v \in H^{2,2}((-\tfrac{L}{2}, \tfrac{L}{2})) \,:\, v_x(-\tfrac{L}{2}) = v_x(\tfrac{L}{2}) = 0\} \subset_{\text{dense}} X \longrightarrow X$$

with $X := L^2((-\frac{L}{2}, \frac{L}{2}))$.

We first state the definition of solutions for the problem (1).