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ON THE NUMBER OF DETERMINING NODES FOR THE GENERALIZED GINZBURG-LANDAU EQUATION

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Abstract In this paper, we obtain that the number of determining nodes for the generalized Ginzburg-Landau equation is two closely points, as a consequence, an upper bound for the winding number of stationary is established in terms of the parameters in the equation. It is also proven that the fractal dimension of the set of stationary solution is less than or equal to 4.

Key Words Generalized Ginzburg-Landau equation; determining nodes; winding number; stationary solutions.

Classification 35K22.

1. Introduction

In the paper [1], we consider the following generalized Ginzburg-Landau equation

$$\begin{aligned}\partial_t u + \nu u_x =& \chi u + (\gamma_r + i\gamma_i)u_{xx} - (\beta_r + i\beta_i)|u|^2 u - (\delta_r + i\delta_i)|u|^4 u \\ &- (\lambda_r + i\lambda_i)|u|^2 u_x - (\mu_r + i\mu_i)u^2 \overline{u}_x, \quad x \in \mathcal{R}^1, \quad t > 0 \end{aligned} \tag{1}$$

where—indicates the complex conjugation, γ_r, δ_r are positive constants, $\nu, \chi, \gamma_i, \beta_r, \beta_i, \delta_i, \lambda_r, \lambda_i, \mu_r, \mu_i$ are real constants. (1) is given by H.R. Brand and R.J. Deissler in [2], (1) is a generalized form of the usual Ginzburg-Landau equation which is obtained in fluid dynamics. We consider (1) with the periodic boundary condition:

$$u(x,t) = u(x+1,t), \quad x \in \mathcal{R}^1, \quad t \geq 0 \tag{2}$$

and the initial condition:

$$u(x,0) = u_0(x), \quad x \in \mathcal{R}^1 \tag{3}$$

In [1], we have obtained the existence of the unique global solution and finite dimensional global attractor in $V_1 = H^1_{\text{per}}[0, L]$ for the problem of (1)—(3) when

$$\gamma_r, \delta_r > 0, \quad 4\delta_r\gamma_r > (\lambda_i - \mu_i)^2 \tag{*}$$

is satisfied.

Our interest is the long time behavior of solutions, certainly, the global attractor describes some information of the long time behavior of solutions, but is too rough. So, in this paper, we are looking for the sets of points which completely determine the long time behavior of solutions, that is, we say that a (finite) set $E \subset [0,1]$ is a set of determining nodes if

$$\lim_{t\to\infty} |u_1(x,t) - u_2(x,t)| = 0, \quad x \in E$$

implies that

$$\lim_{t\to\infty} |u_1(x,t) - u_2(x,t)| = 0, \quad x \in \mathcal{R}^1$$

where u_1 and u_2 are any two solutions.

2. Determining Nodes

The notion of determining nodes was first introduced by Foias and Temam (see [3]) for the Navier-Stokes equations. In [4], Kukavica has shown that the cubic Ginzburg-Landau equation has only two determining nodes. Now we generalize his results to the generalized Ginzburg-Landau equation. First, we introduce some notions.

$$H = L^2_{\text{per}}[0,1] = \{u \in L^2[0,1], u(x+1) = u(x)\}$$
$$V_1 = H^1_{\text{per}}[0,1] = \{u : u \in H, u_x \in H\}$$

where $(.,.)$, $|\cdot|_0$ denote the inner product and norm in H respectively, the norm in V_1 is defined as

$$|u|_1^2 = |u|_V^2 = |u|_0^2 + |u_x|_0^2$$

Theorem 2.1 *Under the condition of* $(*)$, $u_0 \in V_1$, *there exists a postive constant* α_1 *which depends on the coefficients of* (1), *such that, if* $x_1 < x_2$ *then* $d = x_2 - x_1 < \alpha_1$, *and* $\gamma_r > 2d^2\beta_1$ (β_1 *can be seen in the proof*) *and*

$$\lim_{t\to\infty} |u_1(x_i,t) - u(x_i,t)| = 0, \quad i = 1,2 \tag{4}$$

where u_1 *and* u_2 *are two solutions, then*

$$\lim_{t\to\infty} |u_1(x,t) - u_2(x,t)| = 0, \quad \forall x \in \mathcal{R}^1$$