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Dynamic Law of Physical Motion and Potential-Descending Principle

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Abstract. The main objectives of this paper are five-fold. The first is to introduce a general dynamic law for all physical motion systems, based on a new variational principle with constraint-infinitesimals. The second is to postulate the potential-descending principle (PDP). We show that PDP is a more fundamental principle than the first and second laws in thermodynamics, and gives rise to dynamical equations for non-equilibrium systems. The third is to demonstrate that the PDP is the first principle to describe irreversibility of all thermodynamic systems, with thermodynamic potential as the basic physical quantity, rather than entropy. The fourth objective is to examine the problems faced by the Boltzmann equation. We show that the Boltzmann is not a physical law, is created as a mathematical model to obey the entropy-increasing principle (for dilute gases), and consequently is unable to faithfully describe Nature. The fifth objective is to prove an orthogonal-decomposition theorem and a theorem on variation with constraint-infinitesimals, providing the needed mathematical foundations of the dynamical law of physical motion.

AMS subject classifications: 82B05, 82B10, B2C10, 82C40, 35Q20, 35Q82

Key words: Dynamical law of physical motion, potential-descending principle, statistical physics, thermodynamics, entropy, irreversible processes, Boltzmann equation, orthogonal-decomposition theorem, variation with constraint-infinitesimals.

1 Introduction

The heart of physics is to seek experimentally verifiable, fundamental laws and principles of Nature. In this process, physical concepts and theories are transformed into mathematical models and the predictions derived from these models can be verified experimentally and conform to reality. In their mathematical form, the physical laws are often can be expressed as mathematical equations:

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physical laws = mathematical equations.

Among most important physical laws are 1) the laws for fundamental interactions —- gravity, electromagnetism, weak and strong —- and 2) the laws for motion dynamics. Modern theory of fundamental interactions is based on the field theoretical point of view; see among many others [4] and their references therein.

The focus of this paper is on dynamical laws of physical motion systems. According to their scales, the physical motion systems include 1) classical mechanical systems, describing planetary scale motion, 2) quantum systems for particles in the micro level, 3) fluid mechanics systems, macroscopic description of fluid motion, 4) astrophysical systems for astronomical objects, and 5) statistical systems, relating microscopic properties of individual particles to the macroscopic properties.

The main objectives of this paper are 1) to introduce a general dynamic law for all physical motion systems, 2) to postulate the potential-descending principle (PDP), 3) to demonstrate that the potential-descending principle is the fundamental principle to describe irreversibility of all thermodynamic systems, 4) to indicate the problems faced by the Boltzmann equation, and 5) to prove an orthogonal-decomposition theorem and a theorem on variation with constraint-infinitesimals. Hereafter we now give a brief description the main ingredients of this paper.

Dynamic law of physical motion

FIRST, for each isolated physical motion system, there are a set of state functions $u=(u_1,\cdots,u_N)$, describing the states of the system, and a potential functional $F(u)$, representing a certain form of energy intrinsic to the underlying physical system. Then it is physically clear that the rate of change of the state functions du/dt should equal to the driving force derived from the potential functional F. More precisely, we postulate the following *dynamical law of physical motion:*

$$\frac{\mathrm{d}u}{\mathrm{d}t}=-A\delta_{\mathcal{L}}F(u), \tag{1.1}$$

where A is the coefficient matrix, and $-\delta_{\mathcal{L}}F(u)$ is the variation with constraint infinitesimals, representing the driving force of the physical motion system, and $\mathcal{L}$ is a differential operator representing the infinitesimal constraint.

SECOND, we shall demonstrate that proper constraints should be imposed on the infinitesimals (variation elements) for the variation of the energy functional F. These constraints can be considered as generalized energy and/or momentum conservation laws for the infinitesimal variation elements.

The variation under constraint infinitesimals is motivated in part by the recent work of the authors on the principle of interaction dynamics (PID) for the four fundamental interactions, which was required by the dark energy and dark matter phenomena, the Higgs fields, and the quark confinement; see [4]. Basically, PID takes the variation of the Lagrangian actions for the four interactions, under energy-momentum conservation constraints.