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Power Concavity for Doubly Nonlinear Parabolic Equations

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Abstract. We prove the concavity of the power of a solution is preserved for a class of doubly nonlinear parabolic equation, which is a well-known feature in some particular cases such as the porous medium equation or the parabolic p-Laplace equation.

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1 Introduction and main results

In this paper, the geometric quantity preserved in a kinds of double nonlinearity equations is concerned, which may give us the convexity of level sets of the solution.

$$u_t = \nabla\left[|\nabla(|u|^{m-1}u)|^{p-2}\nabla(|u|^{m-1}u)\right], \tag{1.1}$$

where for some $n \geq 2$, $t \in [0,T]$ for some $T < \infty$, and $m > 1$, $m(p-1) > 1$.

Eq. (1.1) has been extensively studied, see [2, 12, 14–16, 29] and their references; for a survey see [12]. In particular, (1.1) has a double nonlinearity as follows:

(a) For $p = 2$, it is the porous medium equation

$$u_t = \triangle u^m, \tag{1.2}$$

(b) and for $m = 1$ which corresponds to the parabolic p-Laplace equation

$$u_t = \nabla(|\nabla u|^{p-2}\nabla u). \tag{1.3}$$

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These two limit cases are prototypes for the main features presented by the solutions of (1.1) and are extensively studied in the literature (see, e.g., [3,9,12,19,25,28] for the porous medium equation and [8,12,13,23,27] for the p-Laplacian).

The geometric quantity preserved in various equations that gives us the convexity of level sets of the solution, with such as idea (1.2) and (1.3) were studied in [9] and [23], respectively. We will be concerned with the geometric quantity preserved of the nonnegative density u for the following Cauchy problem.

$$\begin{cases} u_t = \nabla\left[|\nabla(|u|^{m-1}u)|^{p-2}\nabla(|u|^{m-1}u)\right], & \mathbb{R}^n\times(0,T],\\ u(x,0)=u_0(x), & \mathbb{R}^n. \end{cases} \tag{1.4}$$

We consider the behavior of the solution u in a neighborhood of the free boundary by considering a pressure v

$$u=\left(\frac{v}{\alpha}\right)^{\frac{p-1}{m(p-1)-1}},$$

where α is a constant to be determined, we then have a equation

$$v_t=\left(\frac{1}{\alpha}\right)^{p-1}\left(\frac{m(p-1)}{m(p-1)-1}\right)^{p-1}\frac{m(p-1)-1}{p-1}\left[\nabla(|\nabla v|^{p-2}\nabla v)+\frac{p-1}{m(p-1)-1}|\nabla v|^p\right],$$

we chose a number α such that

$$\left(\frac{1}{\alpha}\right)^{p-1}\left(\frac{m(p-1)}{m(p-1)-1}\right)^{p-1}\frac{m(p-1)-1}{p-1}=1.$$

Thus, v satisfies

$$\begin{cases} v_t=v\nabla\left(|\nabla v|^{p-2}\nabla v\right)+\frac{p-1}{m(p-1)-1}|\nabla v|^p, & \mathbb{R}^n\times(0,T],\\ v(x,0)=v_0(x), & \mathbb{R}^n. \end{cases} \tag{1.5}$$

Remark 1.1.

- If $m=1$, i.e., $\alpha=\left(\frac{p-1}{p-2}\right)^{\frac{p-2}{p-1}}$, $u=\frac{p-2}{p-1}v^{\frac{p-1}{p-2}}$, then (1.5) becomes

$$v_t=v\nabla\left[|\nabla v|^{p-2}\nabla v\right]+\frac{p-1}{p-2}|\nabla v|^p,$$

which is studied in [8,12,13,23,27].

- If $p=2$, i.e., $\alpha=m$, $u=\left(\frac{v}{\alpha}\right)^{\frac{1}{m-1}}$, then (1.5) becomes

$$v_t=v\triangle v+\frac{1}{m-1}|\nabla v|^2,$$

which is studied in [3,9,12,19,25,28].